Introducing the Concept of Integral via Experimentation, Discovery, and Writing

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This is a laboratory, experimentation, and writing project designed to introduce and reinforce the concept of integration. The technology used in this project is a sonic distance probe, a Texas Instruments CBL, and a TI-83 calculator equipped with a data collection program suitable to the sonic probe and CBL. The student rolls a ball down a wooden ramp, using the probe-CBL-Calculator combination to collect data on the position and velocity of the ball as functions of time.

The central question asked in the project is: How can we construct the position graph from the velocity graph? The students are guided in seeing that this would be easy to do if the velocity were constant8212;an example of constant velocity motion is given to them, and they work with it sufficiently to remind themselves of how constant velocity problems work.

The objective then is to see how to use these same concepts to deal with motion the velocity of which is not constant. They have data for the velocity of their ball as a function of time; it will be a linear function, more or less, and the experiments I and my students have done successfully produce graphs that are convincingly linear. A TI-83 calculator can fit a linear equation to such data. Using Maple, I instruct my students to print out a copy of this graph; one may also use the Texas Instruments Graphlink.

At this point, the student is introduced to the idea of breaking the time interval into sections and approximating the distance traveled by using the velocity of the ball at the beginning and the end of each section. Thus we introduce left hand sums and right hand sums. The student is guiding in thinking about geometrical interpretations of these sums, and I found that with a few hints, they think of the area of rectangles. They are instructed to draw in these rectangles. Thus they are guided to see that distance traveled can be interpreted as area under a velocity curve. Since the velocity curve, in this case, is very simple, and in fact just a straight line, the student is asked to find the area under the curve geometrically, as a function of time. All this amounts to is using the formula for the area of a trapezoid; however, I find it is tremendously convincing for the students to see for themselves that the graph of the function thus produced does in fact match the original graph of the data of distance as a function of time.

I have students do this project in pairs. They are expected to produce a three to four page paper in which they explain carefully the problem they are working to solve, the methods that they used, and their results. I employed this project this past January to introduce the concept of integration to my Calculus II students and I found it quite successful; I will have several examples of work produced by students to present.