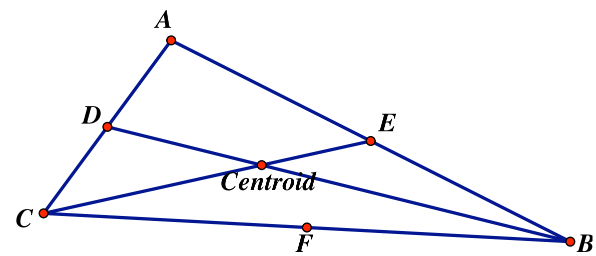
Centers of Triangles

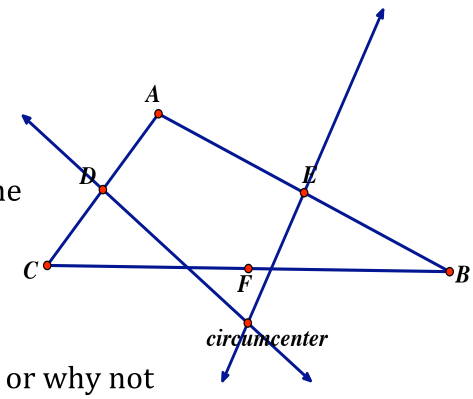
Objective: To construct the four centers of a triangle: the centroid (medians), circumcenter (perpendicular bisectors of the sides), incenter (angle bisectors), and orthocenter (altitudes).



1. **Centroid**
   NY Standards: G.G. 43
   a. Start GSP or open a new sketch.
   b. Construct a triangle.
   c. Label the vertices A, B, and C.
   d. Select two sides, under the **Construct** menu, chose **Midpoints.** Label the midpoints D, E, and later F.
   e. Connect the midpoints to the opposite vertex – this constructs the medians.
   f. Locate the point of intersection. Label this point **Centroid.** Verify that the third median goes through this point.
   g. The centroid is called the center of mass of the triangle. If you were to cut out this triangle, you could balance it on the head of a pin placed at the centroid.
   h. Can the centroid be outside the triangle? Why or why not (explain in a text box)?
   i. Explain (in a new text box) as best you can why you think the intersection of the lines you've constructed is the centers of mass (centroid). You might do a bit of online searching to see what discussions there are about this topic.

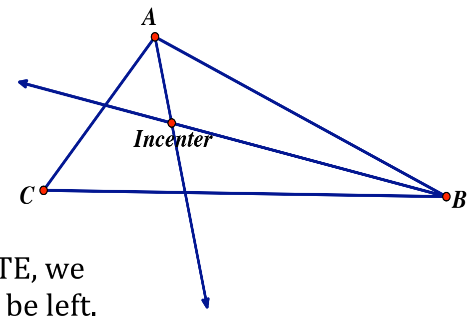
2. **Circumcenter**
   a. Under the **File** menu, choose **Document Options.** In the dialog box, there is a drop down menu titled, **Add Page.** Choose **Blank Page** four times (at the bottom of the GSP window, you should see tabs numbered 1, 2, 3, 4, and 5). Click on the 1 in the dialog box (for page 1) and in the Page Name text box, type Centroid. Click on the 2 in the dialog box (for page 2) and in the Page Name text box, type Circumcenter (do likewise for pages 3 and 4; we'll name 5 later). Click on **Ok**.
   b. Choose the tab labeled Centroid. Select all of the objects by going to the **Edit** menu, choose **Select All** and under the **Edit** menu choose **Copy**. Now, click on the tab labeled Circumcenter.
   c. In the **Edit** menu, choose **Paste**. You should see the same triangle that is on the Centroid page.
   d. For the triangle on the Circumcenter tab, **hide** the three medians (select the lines and go to the **Display** menu, **Hide Objects**) and centroid point. DO NOT DELELTE, we will need these later! The triangle and three midpoints should be left showing.
   e. Through the midpoints, construct perpendicular bisectors to two sides.

f. Locate the point of intersection. Label this point **Circumcenter**. Verify that the third perpendicular bisector goes through this point.
g. To see why this is called the circumcenter, select the circumcenter and point A. Under the **Construct** menu, choose **Circle by Center+Point**. You will see that the circumcenter is the center of the circle that circumscribes the triangle.

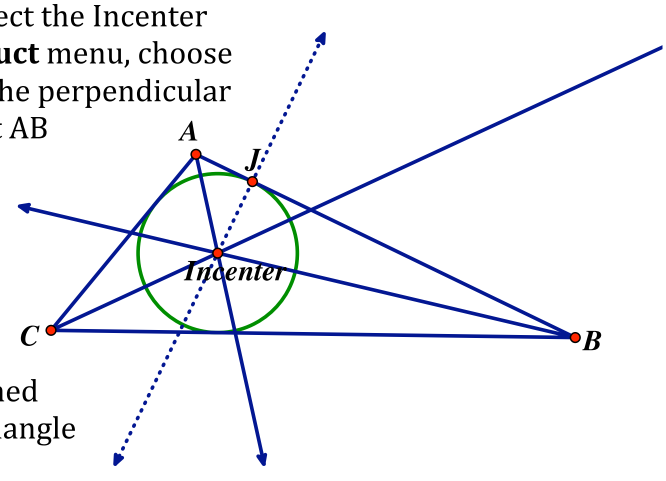


h. Can the circumcenter be outside the triangle? Why or why not (explain in a text box)?
i. Explain (in a text box) as best you can why you think the intersection of the lines you've constructed form the center of a circle that circumscribe the triangle. You might do a bit of online searching to see what discussions there are about this topic.

3. **Incenter**
   a. Unhide all of the objects in the Circumcenter tab (under the **Display** menu and choose, **Show All Hidden**).
   b. Under the **Edit** menu, choose **Undo Construct Circle.**
   c. Under the **Edit** menu, choose **Select All** and in the **Edit** menu, choose **Copy.**
   d. Go to the tab labeled Incenter and under the **Edit** menu, choose **Paste.**
   e. Hide all of the lines and points. DO NOT DELELTE, we will need these later! Only triangle ABC should be left.



   f. Select an angle by clicking on the three points in order, then under the **Construct** menu, choose **Angle Bisector.** Repeat this on a second angle.
   g. Locate the point of intersection. Label this point **Incenter**. Verify that the third angle bisector goes through this point.
   h. To see why this is called the Incenter, select the Incenter point and segment AB. Under the **Construct** menu, choose **Perpendicular**. Locate the point where the perpendicular line from the Incenter intersects segment AB (the perpendicular line is the dotted line in the figure). The point of intersection is labeled J.
   i. Select the Incenter and the intersection point J, then under the **Construct** menu choose **Circle by Center+Point.** You will see a circle that is completely contained inside the triangle and each side of the triangle is tangent to the circle.



   j. Will the Incenter be outside the triangle? Why or why not (explain in a text box)?

k. Explain (in a new text box) as best you can why you think the intersection of the lines you've constructed form the center of a circle that is inscribed in the triangle. You might do a bit of online searching to see what discussions there are about this topic.

4. **Orthocenter**

a. Unhide all of the objects in the Incenter tab (under the **Display** menu, choose **Show All Hidden**).

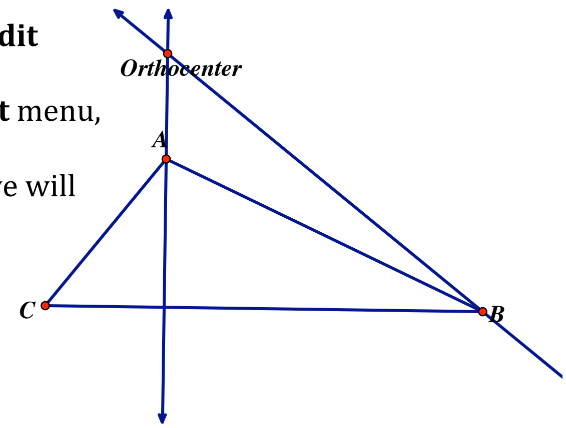
b. Under the **Edit** menu, choose **Undo Construct Circle, Undo Construct Intersection,** and **Undo Construct Perpendicular.**

c. Under the **Edit** menu choose **Select All**, and in the **Edit** menu choose **Copy**.

d. Go to the tab labeled Orthocenter and under the **Edit** menu, choose **Paste**.

e. Hide all of the lines and points. DO NOT DELELTE, we will need these later! Only triangle ABC should be left.

f. Select vertex A and segment BC, the opposite side. Under the **Construct** menu, choose **Perpendicular.** Repeat for vertex B and segment AC.



g. Locate the point of intersection. Label this point the orthocenter. Verify that the third altitude goes through this point.

h. Can the orthocenter be outside the triangle? Why or why not (explain in a text box)?

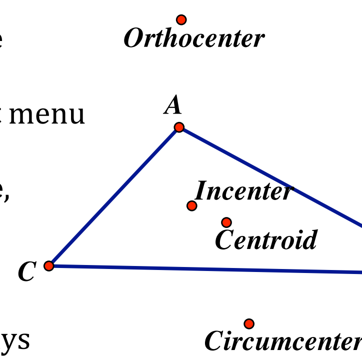
5. Euler Segment

This exploration uses the triangle centers constructed above.

a. Unhide all the objects in the Orthocenter tab (under the **Display** menu, choose **Show All Hidden**).

b. Under the **Edit** menu, choose **Select All**, and in the **Edit** menu choose **Copy.**

c. Go to page 5 (you can name it Euler Segment; Go to **File**, choose Document Options, click on the 5, and in the Page Name text box type, Euler Segment). Under the **File** menu, choose **Paste.**



d. Hide all the lines. Continue until there are no lines or rays left sticking out from the triangle. Now select the three medians and the three midpoints of the sides. Hide them, as well.

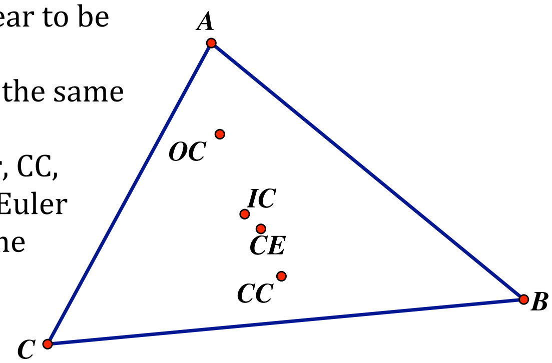
e. You should now have only the four centers (centroid, circumcenter, incenter and orthocenter) and the triangle remaining.

f. To make the centers easier to see re-label the centroid as CE, the circumcenter as CC, the incenter as IC and the orthocenter as OC.

g. Drag a vertex of your triangle around. Which centers appear to be collinear (answer in a text box)?

h. Are these centers collinear for any triangle (make various measurements to determine which type of triangle you have)?

i. Are there any triangles where all four centers appear to be collinear?
j. Are there any triangles where the four centers are the same point?
k. Connect the orthocenter, OC, and the circumcenter, CC, of your triangle with a segment. This is called the Euler Segment. Now drag around one of the vertices of the triangle. What do you notice?



l. Draw a horizontal segment across the top of your sketch. Select that segment and the top vertex of your triangle. Under the **Edit** menu, choose **Merge Point to Segment.**
m. Under the **Edit** menu, chose **Action Button** and then slide to the right to **Animation.** A dialog box should appear. Select **medium** as the speed. Then click the **OK** button. The Animate Point button should appear on your sketch.
n. Click on the Animate Point button and observe what happens. Click the button again to stop the animation.