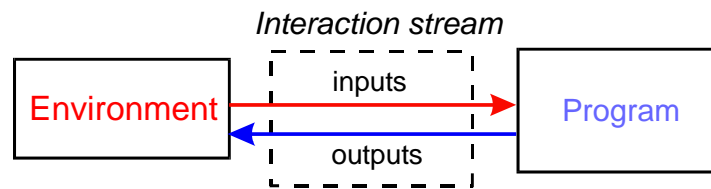


Streams and coinduction



Sources:

- Goldin-Wegner (at www.engr.uconn.edu/~dgg)
- Barwise-Moss, *Vicious Circles*, 1996
- Finsler, 1920s (NWF sets)
- Peano, 1889 (induction)

Inductively defined sets

- (i) 0 is a natural number;
- (ii) Every $n \in \mathbf{N}$ has a unique successor, n' ;
- (iii) i and ii are the only ways to obtain a natural number (Peano)
- Set of expressions using numerals, +, and ():
 $expression \rightarrow$
 $numeral \mid$
 $numeral + expression \mid$
 $(expression)$
- The definition of such a set may *use* itself
- But such a set does not *contain* itself

Well-founded sets

- Let the village hair stylist be the person who cuts the hair of everyone who doesn't cut own hair
- Question: Who cuts the hair-stylist's hair?
- Similar Questions: Is there a set of all sets? If so, does it contain itself? Is there a set of all sets that don't contain themselves?
- In classical (well founded) set theory, it is meaningless to say a set belongs or doesn't belong to itself (Well-Foundedness Axiom)

Streams and non well founded sets

- A non-well-founded set may contain itself, directly or indirectly
- *Example:* $A = \{ B, C \}; B = \{ A, D \}$
- Every stream of characters is a character, followed by a stream of characters
- Streams contain streams, which contain streams, which contain streams, ...
- *Ex.:* The set of all bit streams $\{0,1\}^\infty$ consists of any infinite sequence that consists of 0 or 1 followed by a stream

Streams and coinduction

- *Coinduction* is a dual of induction (recursion), lacking a base case
- Coinduction is used to define sets of infinite objects
- The set of streams over an alphabet, expressing ongoing processes such as interaction, is uncountable
- Note: the set of *strings* is countable