Algorithm verification: Sample problems

The problem below, with solution, is a model to help solving problems in assignment 2.

1. *Sum* problem

Find the sum of the elements on an array of numbers.

Iterative solution with assertions:

Sum(A)

> *Precondition: A* is an array of numbers
$i \leftarrow 1$
$y \leftarrow 0$
while $i \leq |A|$
  > Loop invariant: $y = A[1] + A[2] + ... + A[i - 1]$
  $y \leftarrow y + A[i]$
  $i \leftarrow i + 1$
return y
>*Postcondition*: $y = A[1] + A[2] + ... + A[|A|]$

Correctness proof:

We proceed by induction.

1. (Base) The loop invariant holds on the first iteration, because y is 0 and $A[i - 1]$ refers to no element of A.
2. (Induction) If the loop invariant holds on the nth iteration, then it will hold on the $(n + 1)$st, because on each iteration $A[i]$ is added to y, and the element $A[i]$ takes the name $A[i - 1]$ after the incrementing of i.
3. The precondition follows from the loop invariant, because at the beginning of the last iteration, $i = |A| - 1$, and that iteration adds 1 to i and adds $A[|A|]$ to y.
4. Hence at the end of the last iteration, $y = A[1] + A[2] + ... + A[i]$, which is the same assertion as the postcondition, since $i = |A|$.

2. *Last-zero* problem

Problem: Find the location of the rightmost zero in a bit array.
Solution:

Last0(A)
// Pre: |A| > 0 and *A* contains a 0
$i \leftarrow 1$
while $i \leq |A|$
> Loop invariant: y stores location of
> rightmost 0 in $A[1 .. i - 1]$
  If $A[i] = 0$
    $y \leftarrow i$
  $i \leftarrow i + 1$
return y
// Post: y stores location of rightmost 0 in A

Correctness proof:

We proceed by induction. [*To be completed.*]