

Developing Skills, Confidence and Community in Freshmen Mathematics Majors

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Course Manuals*

*Developing Successful Math Majors: A Two Semester Course Sequence
Student Manual and Instructor's Manual* by Thomas Zachariah, Ph.D.,
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Additional Workshop in Mathematics Course Documentation
<http://www.lacteonline.org/Curriculum/jackie/abstract.htm>

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Loyola Marymount University Workshop in Mathematics
Problem Solving Component Topics

First Semester Problem Solving Strategies

Understanding the Problem
Breaking Mind Set
Simplify the Problem
Make a Model/Draw a Figure
Collect and Organize Data/Look for a Pattern
Working Backwards
Persistence
Another Approach
Looking Back
Putting It All Together

Second Semester Problem Solving Strategies

Draw a Diagram
Inductive Approach
Use the Pigeonhole Principle
Pursue Parity
Break the Problem Down by Cases
Special Cases
Fewer Variables
Contradiction
Putting It All Together

Mathematical Writing Skills Topics

Learning to Write Mathematics Activities/Assignments

Writing Samples for Students
Incorrect Math
Use Proper English, Mathematical Terms, and Write in Complete Sentences
Strike a Balance Between Words and Symbols
Honor the Equal Sign
Use Different Letters for Different Things
Define Terms and Notation
Give Reasons
Watch Your Pronouns
Mathematical Topic Paper/Poster

Writing to Learn Mathematics Activities/Assignments

Two Minute Reflections
Mathematical Writing in Other Courses Assignment
Portfolio Writing Assignment

Workshop-Course Peer Review Sheet

Peer Review of _____ 's
by _____

Problem # _____

Due date: _____

- | | | | | |
|----|--|-----|-------|----|
| 1. | Is the answer correct? | YES | MAYBE | NO |
| 2. | Is the mathematical argument correct? | YES | MAYBE | NO |
| 3. | Are there reasons given for each step? | YES | MAYBE | NO |
| | a. Is there enough mathematics or notation in the explanation? | YES | MAYBE | NO |
| | b. Are there enough words of explanation? | YES | MAYBE | NO |
| 4. | Is the mathematical notation good? | YES | MAYBE | NO |
| | a. Is every symbol defined? | YES | MAYBE | NO |
| | b. Is the same symbol only used for equal quantities? | YES | MAYBE | NO |
| | c. Is the equal sign used correctly? | YES | MAYBE | NO |
| 5. | Is the English correct? | YES | MAYBE | NO |
| | a. Are the sentences complete? | YES | MAYBE | NO |
| | b. Do all pronouns have a clear antecedent? | YES | MAYBE | NO |
| | c. Is the spelling correct? | YES | MAYBE | NO |
| 6. | Is the presentation or format neat and clear? | YES | MAYBE | NO |

Explain all MAYBE or NO responses. Refer to specific line numbers.

Workshop in Mathematics - Selected Writing Assignments

Mathematical Writing Assignment: Incorrect Math

Mathematics is the most precise of all subjects. *Every statement you make must be correct.* A solution will be ruined by a single false step such as

$$\frac{a+3}{b+3} = \frac{a}{b} + 1 \quad \text{or} \quad (x+y)^2 = x^2 + y^2 \quad \text{or} \quad (x^5)^2 = x^{25}.$$

If you are not sure about a statement, check it for some special cases. For example, if you suspect that $n^2 > 2^n$, try a few values of n . You will soon see that $5^2 < 2^5$, so the statement is false. (Price [9, Writing Bibliography])

Directions: The following writing samples have been taken from student papers. Rewrite each, being sure to correct any misstatements. Where possible, give improvements to the writing.

- Let $f(x) = 10x - x^2$. Then
 $f(x+h) = 10(x+h) - (x+h)^2$
 $= 10x + 10h - (x^2 + 2xh + h^2)$
 $= 10x + 10h - x^2 - 2xh - h^2$
 $= -x^2 + 10x - 2xh + 10h - h^2$
OR $= x^2 - 10x + 2xh - 10h + h^2$.

Mathematical Writing Assignment: Use Proper English, Mathematical Terms, and Write in Complete Sentences

Good communication requires good English. A correct solution garbled by bad English may be worthless to the reader. The rules of grammar, spelling, and punctuation apply to mathematics. For clear communication, present one idea at a time. Since an idea is expressed by a complete sentence, *Write in complete sentences.* (Price [9, Writing Bibliography]) It is also important to use correct mathematical terms. For example, if you use the term *equation* when referring to an *expression*, the reader will be misled as to what you are referring.

Directions: The following writing samples have been taken from student papers. Rewrite each, being sure to write in complete sentences, and use proper English and mathematical terms. Where possible, give improvements to the writing.

- $2^x = 7$
 $x \log_{10} 2 = \log_{10} 7$
 $x .3010 = .8451$
 $x = 2.81$.

Mathematical Writing Assignment: Strike a Balance Between Words and Symbols

In well written mathematics, there is a balance between words and symbols. A mathematical paper written entirely in words, and a mathematical paper written entirely in symbols is not good. Each can be very difficult to read. A general rule of thumb for most mathematics is that it should be written with about half words and half symbols. Even when using words, try to be accurate and concise. Long-windedness gets confusing.

Directions. Consider the following examples. Decide whether each could be better stated with more words or more symbols. Rewrite each example.

1. If you have a triangle with a right angle and you take the square of the longest side and the square of the other two sides and then add up the squares of the other two sides, that number is the same as the square of the longest side.
3. $x^2 - 2x - 3$
 $(x - 3)(x + 1) = 0$
3, -1

Mathematical Writing Assignment: Honor the Equal Sign

Quantities on either side of an equal sign must be equal. Oranges do not equal apples, numbers do not equal sets. The equal sign has a precise meaning; it is not a punctuation mark such as a dash - . (Price [9, Writing Bibliography])

Directions: Several of the following writing samples have been taken from student papers. Rewrite each, being sure that the equal sign is used properly. Where possible, give improvements to the writing.

1. Problem: Find all roots to the polynomial $p(x) = 4x^2 - 13x + 3$.

$$\begin{aligned}\text{Solution: } p(x) &= 4x^2 - 13x + 3 \\ &= (4x - 1)(x - 3) = 0 \\ &= 3, 1/4.\end{aligned}$$

2. Problem: Let $A = \{x : 2 < x < 4\}$, $B = \{x : x \leq 4\}$, $C = \{x : x > 3\}$. Find $A \cup (B \cap C)$.

$$\begin{aligned}\text{Solution: } \{x : x \leq 4\} \cap \{x : x > 3\} &= \{x : 3 < x \leq 4\} \\ &= \cup \{x : 2 < x < 4\} \\ &= \{x : 2 < x \leq 4\} = (2, 4].\end{aligned}$$

Mathematical Writing Assignment: Use Different Letters for Different Things

A person reading mathematics can become very confused if a single piece of notation is used to represent two (or more) different things within the same section. In fact, using a single piece of notation to represent two different things often will make the mathematics technically incorrect. For example, when writing an equation showing that n is a power of 2, it is incorrect to write $n = 2^n$. It is better to write $n = 2^k$ for some integer k .

Directions: Several of the following writing samples have been taken from student papers. Rewrite each, being sure that notation is used properly. Where possible, give improvements to the writing.

2. **Problem:** Prove that the product of two odd numbers is odd.

Solution: Suppose n, m are odd numbers. Then $n = 2k + 1$ for some integer k and $m = 2k + 1$ for some integer k . So $n \cdot m = (2k + 1)(2k + 1) = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. This has the form of an odd number: $2k + 1$ where $k = k^2 + 2k$. So $n \cdot m$ is odd.

Mathematical Writing Activity: Watch Those Pronouns

When writing mathematical explanations or proofs in paragraph form, students often use pronouns instead of more accurate mathematical terminology. As a result, the writing is unclear. One essential feature of good mathematical writing is its clarity. When proofreading your writing, pay close attention to the occurrence of any pronouns: *it, one, this, that, these, those, they*. If it is not obvious what the pronoun refers to, replace the pronoun with a mathematical term or phrase. Check to see that any remaining pronouns clearly refer back to a specific antecedent. Inserting equation numbers near the left or right margins can be a convenient way to avoid an unclear use of a pronoun if it refers back to an equation.

Directions: In the writing samples below, underline each occurrence of a pronoun. Then re-write the solution, replacing pronouns as needed to increase clarity. Make any other improvements to the writing as well.

1. **Problem:** Find the slope of the line: $3x - 7y + 5 = 0$.

Solution: To find it we solve it for y :

$$\begin{aligned} 3x - 7y + 5 &= 0 \\ 7y &= 3 + 5 \\ y &= \frac{3}{7}x + \frac{5}{7} \end{aligned}$$

So it gives us $m = \frac{3}{7}$.

Mathematical Writing in Other Courses Assignment (This assignment may be given several times during the semester.)

Select a quiz or homework problem that you got wrong in one of your other mathematics courses this semester. **Pick something with substance.** Identify clearly the class and the problem. Give a critique of the writing and tell what mistake you made. Then rewrite it, correcting the mistake and making as many other improvements in the writing as you can.

Portfolio Writing Assignment (Given at the end of each semester)

Select 3 pieces of your mathematical writing, either original problem solving solutions, write-ups, or re-writes. Your selection should show the scope of your improvements in writing mathematics this semester. Write in some detail why you chose the 3 pieces and how it is that your selections demonstrate your improvement.

Workshop in Mathematics - Selection of First Semester Problems/Strategies (see [1], [14], [20], [28])

Understand the Problem

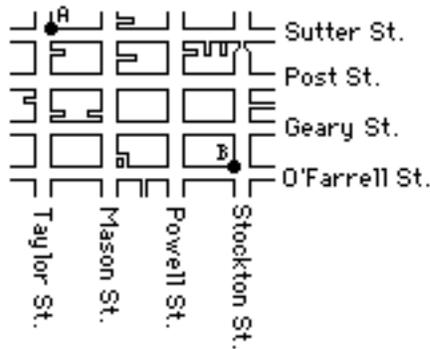
1. You have two square remnants of imported fabric; the side of one square is 3 yards and the side of the other is 4 yards. Suppose you cut each of the squares into two pieces and arrange the four pieces into one larger square. What are the dimensions of the resulting square?
7. To number the pages of a bulky volume, the printer used 2989 digits. How many pages has the volume?

Breaking Mind Set

1. An Indian was tracking a bear, which had broken into her shed. She went one mile south, 1 mile east, 1 mile north and she found herself back where she started. What color was the bear?
11. Cucumbers are assumed, for present purposes, to be a substance that is 99% water by weight. If 500 pounds of cucumbers are allowed to stand overnight, and if the partially evaporated substance that remains in the morning is 98% water, how much is the morning weight?

Simplify the Problem

1. What single three-dimensional shape will go through all three of the holes? 
2. Compute $111,111,111^2$. Do not use a calculator or computer.
3. Melissa lives at the YWCA (point A) and works at Macy's (point B) as shown in the map below. She usually walks to work. How many different routes can Melissa take if she doesn't backtrack - that is, if she always travels toward her destination?



- In how many zeros does 1,000,000! (one million factorial) end?

Make a Model/Draw a Figure

- (pre-test) The 4 volumes of a certain math encyclopedia are standing in order (left to right) on a bookshelf. Each book is $\frac{5}{4}$ " thick counting the covers, each of which are $\frac{1}{8}$ " thick. If a bookworm eats from the first page of volume 1 straight through to the last page of volume 4 including the intervening covers, how many inches has it devoured its way through?
- A college and the local movie theater are ten blocks apart. On his way from the college to the movie, Joe passes a sports arena first and then his dorm. It is a block farther from the college to the arena than from the arena to the dorm, and it is three blocks farther from his dorm to the theater than from the arena to the dorm. How far is it from the college to his dorm?

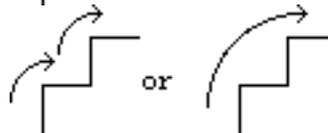
Collect and Organize Data/Look for a Pattern

- (pre-test) Count the number of ways a flight of 10 stairs can be climbed according to the following rule: either one or two steps can be taken at one time.
Examples:

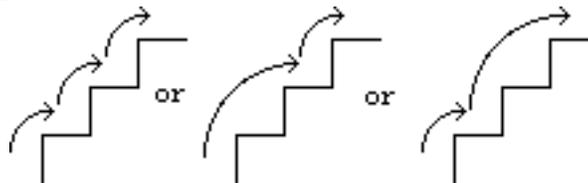
Only one way to climb one stair:



Two ways to climb two stairs:



Three ways to climb three stairs:



Working Backwards

- (pre-test) Renee rented a booth at the Renaissance Faire to sell handmade sweaters. The first customer bought half of Renee's sweaters and half of a sweater. The second customer bought half of the remaining sweaters and half a sweater. When the third and last customer had purchased half of Renee's remaining sweaters and half a sweater, Renee had only 3 sweaters left. Knowing

that Renee would not cut apart one of the sweaters, how were the three sales possible? How many sweaters did Renee sell?

Persistence

2. A woman engineer met a woman mathematician while working on the space shuttle. One day they were discussing how they managed to bring up their children while working full time in the space program. When the engineer asked the mathematician how old her three children were, the mathematician said, "The product of their ages is 72 and the sum of their ages is the same as the space shuttle number." The engineer stated that she needed more information. The mathematician replied, "My oldest child is named Hypatia after the woman mathematician, Hypatia of Alexandria." With that bit of additional information even you can figure out the ages of the mathematician's children.

Looking Back

2. If 5 lines intersect in a single point, how many pairs of vertical angles are formed? Can you generalize? Justify.
3. What is the sum of the angles in any convex hexagon? Can you generalize? Can you justify your answer in more than 1 way?

COMMENT: The first semester problems are intentionally quite elementary, and often recreational in flavor. We do not want students to be frustrated by mathematical content they have not yet mastered. Instead we want them to concentrate on the problem solving PROCESS, thereby becoming more aware of and able to monitor their own approaches, strengths, and weaknesses.

Workshop in Mathematics - Selection of Second Semester Problems/Strategies (see [14], [20], [28])

Draw a Diagram

1. (pretest) Let A and B be given real numbers. Suppose that for all positive values of C the root(s) of the equation $Ax^2 + Bx + C = 0$ are real positive numbers. Present an argument to show that A must equal zero.

Pursue Parity

1. Suppose a classroom has 25 students seated in desks in a square 5 x 5 array. The teacher wants to alter the seating by having every student move to an adjacent seat (just ahead, just behind, on the left, or on the right). Is such a move possible?

Special Cases

3. Let $P(x)$ and $Q(x)$ be two polynomials with "reversed" coefficients. For example, $P(x) = 7x^3 - 5x^2 + 3x - 1/2$ and $Q(x) = (-1/2)x^3 + 3x^2 - 5x + 7$.

What is the relationship between the roots of $P(x)$ and those of $Q(x)$? Prove your answer.

Fewer Variables

2. Show that if x , y , and z are greater than 0, then $(x^2+1)(y^2+1)(z^2+1)/xyz \geq 8$.

Inductive Approach

2. Find at least 3 approaches to discovering and/or proving a formula for the sum of the first n odd integers.
5. Consider the data: $1 + 8 + 27 + 64 = 100$. What does it suggest? State the "result" as a formula and in words. Is it always true?

Contradiction

6. *Prove or disprove:* There exist a set of numbers using each of the ten digits exactly once so that the sum of the numbers is exactly 100.
7. *Prove or disprove:* Given that a , b , c are odd integers, prove that the equation $ax^2 + bx + c = 0$ cannot have a rational root.

Putting It All Together

2. A knight is placed in the center of a square board of 25 squares. It is possible to move in such a way so as to visit each square once and only once? How should it go?
4. Solve the equation $(he)^2 = she$ where, s , h , and e represent distinct digits.
6. Is it true that for all positive real numbers x , that $2\sqrt{x} \leq x + 1$?
9. If c is an odd integer, can there be an integer n such that $n^2 + n - c = 0$?

Two Minute Writing-to-Learn Reflections

I ask students to reflect on the four "Simplify the Problem" strategy problems we solved (see page 7) and to write a few sentences about what they want to remember from that day's class. (All *italics* below are my emphasis.)

Student #1

I never would have thought of trying smaller numbers to solve a larger one. It seemed too tedious and I would expect it to take longer than tackling it by itself. After being forced to use smaller numbers, I do see it as being much simpler in the end. The problem about computing $111,111,111^2$ made me realize this. I'm glad I did get to become exposed to these new methods because it does give me a chance to change: something I rarely do.

Student #2

On $111,111,111^2$. Taking small cases or simpler cases was very beneficial in helping to solve the problem. It helped to see the pattern there was.

I also learned about Pascal's Triangle and how it can be used to express things like $(a + b)^4$.

Student #3

- Break down one large problem into several sub-problems.
- Solve the problem on a smaller scale, use smaller numbers.
- Draw pictures and label them clearly. Visualization is the key.
- *Be organized. Don't lose data because you're messy.*
- Pascal's Triangle is a handy tool.

Student #4

We had to break the larger problems down by simplifying the original. *This is something I often forget to do. It refreshed Pascal's Triangle for me. It forced me to organize my data which I still need to work on. Usually I have trouble finding related problems.*

Student #5

Before today I thought simplifying was factoring something. Now I feel like I've learned that simplifying is not just factoring, but also doing only a part of a problem or reducing the size of the numbers involved in the problem.

Student #6

Something I have learned today is how to apply Pascal's Triangle to a real life problem. *I also learned that 3 minds are better than one.*

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