Introduction to Proofs as A Survey Course in Mathematics

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Overarching themes give segues from one subfield to the next.

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3. **Number Theory**
   $\sqrt{\text{prime}}$ is irrational. Algebraic vs. transcendental numbers. Diophantine equations. Fermat and Wiles. Solutions to quadratics, cubics, and quartic polynomials.

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7. **Complex Analysis**
   Polar representation and complex number arithmetic. Single- and multi-valued functions. Cauchy-Riemann equations and harmonic functions. Power series for analytic functions.

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At Butler: from **30** (the four-cohort total at January, 2011) to **60** (the four-cohort total today).

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The survey course helps recognize this problematic situation early. It identifies students who should not continue, and THEY also realize math is a bad fit. The course instructor can then successfully counsel and advise (it's not ALL about more numbers): find a different major that fits.

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3. Students suddenly want to take upper-level courses—each subfield study gives an introduction to basic definitions and student confidence.
4. Students report they see and enjoy mathematics' beauty, how "cool" it is, and how it has developed historically—across the survey of subfields. Math is better than they realized from calculus—it has a sense of discovery and many unsolved problems. This good news is previously unrealized by almost all these students.

To read more: WJ and McAllister, Alex, *A Survey Transition Course*, PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, Volume 22, Issue 1, 2012.

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