

Projective Geometry  
*for the Liberal Arts Mathematics  
Class*

DONNA A. DIETZ



Projective Geometry....

Not just for Math Majors anymore!

# Projective Geometry....

Not just for Math Majors anymore!

ages 5 - 115

# The Spot-it Deck:

55 Cards, each with 8 symbols.

Any set of two cards has exactly one symbol in common.

On the next slide, I will show two *Spot it!* cards.  
Find the common symbol.















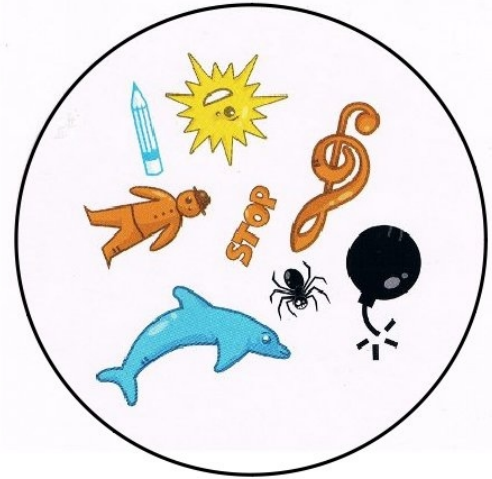
# Learning Objectives:

*(why Mathematicians/teachers like it)*

- Working with Finite Geometries (point/line axiomatic systems)
- Deeper understanding of modular arithmetic
- Probability
- Combinatorics
- Finite Projective Planes / Affine Planes
- Duality
- Axioms
- Single/Group problem solving skills

# What the students like:

- Hands on! (scissors, drawing, arranging things)
- On-line apps to use in and out of classroom
- It is a game.
- Productive group work
- Visually appealing
- Formulas come later, after they make sense.
- This topic is brand new to nearly everyone, so nobody feels like “the only one” who hasn't done this before.









# Please Read My Upcoming Paper....



- Cambridge Undergraduate Mathematics Journal.
- Since 1939, not quite every year....
- Paul Erdos, Martin Gardner, John Conway, Hardy, Penrose... and soon... ME!

[http:// www. donnadietz. com](http://www.donnadietz.com)







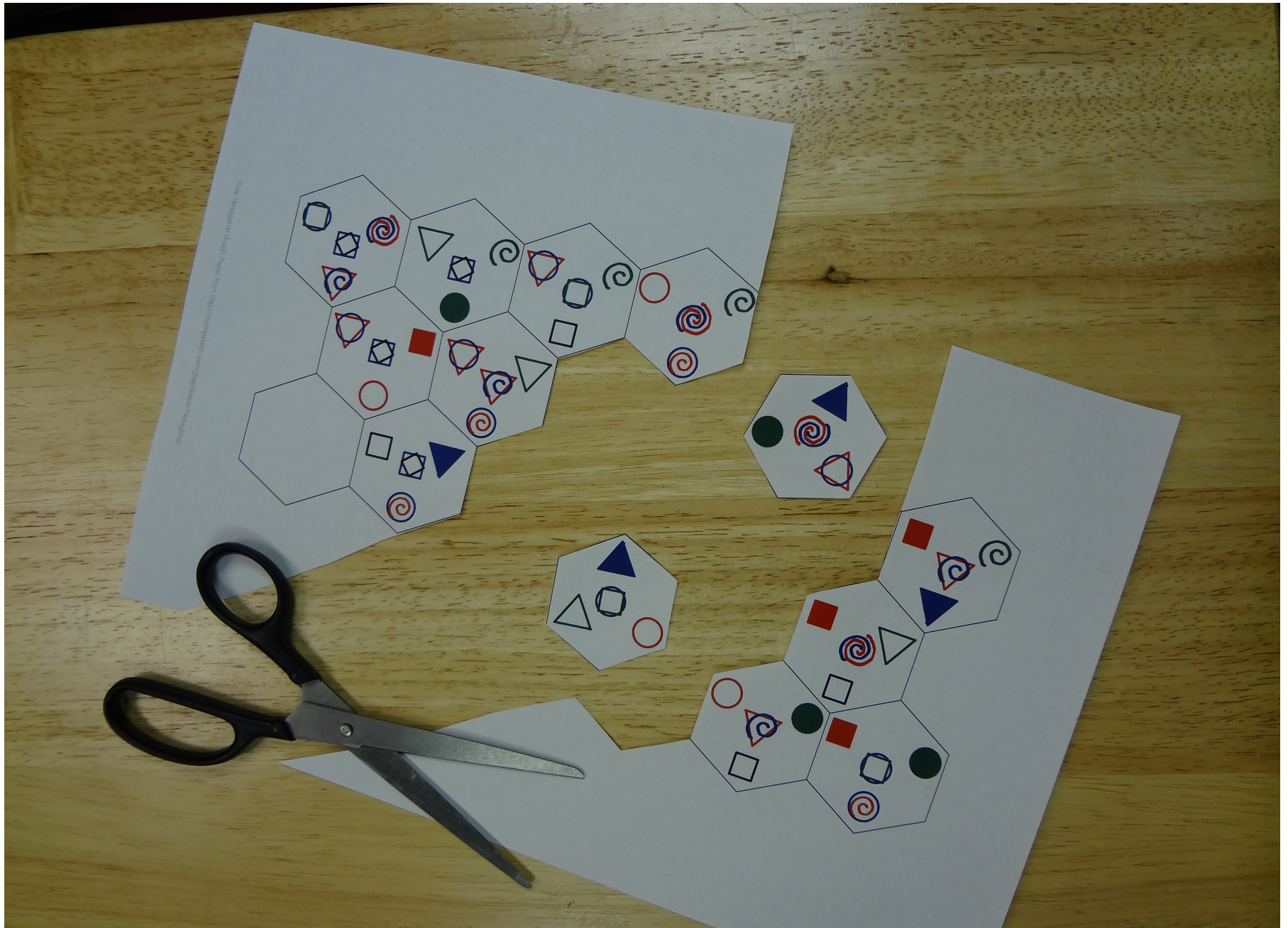
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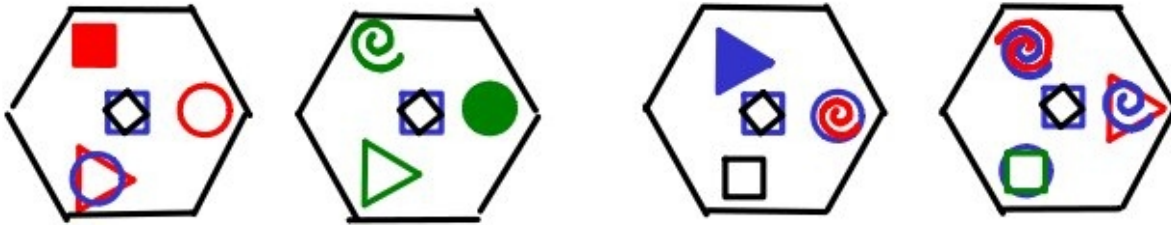
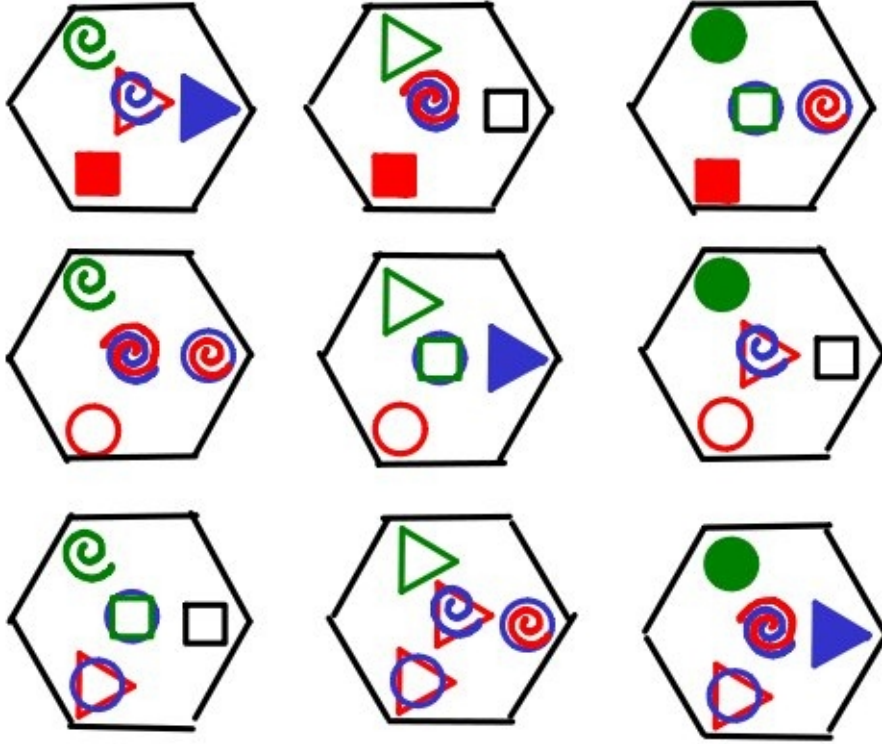
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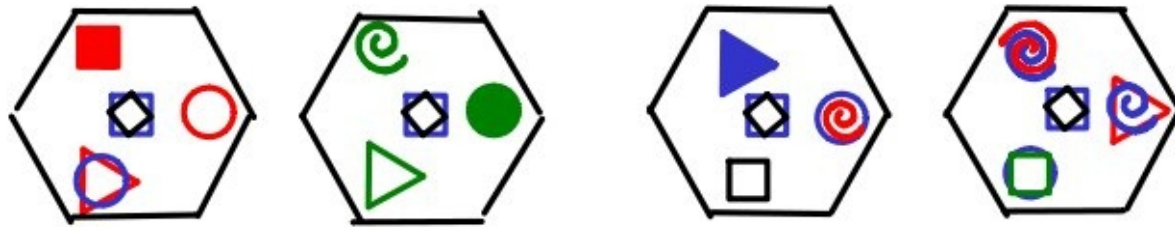
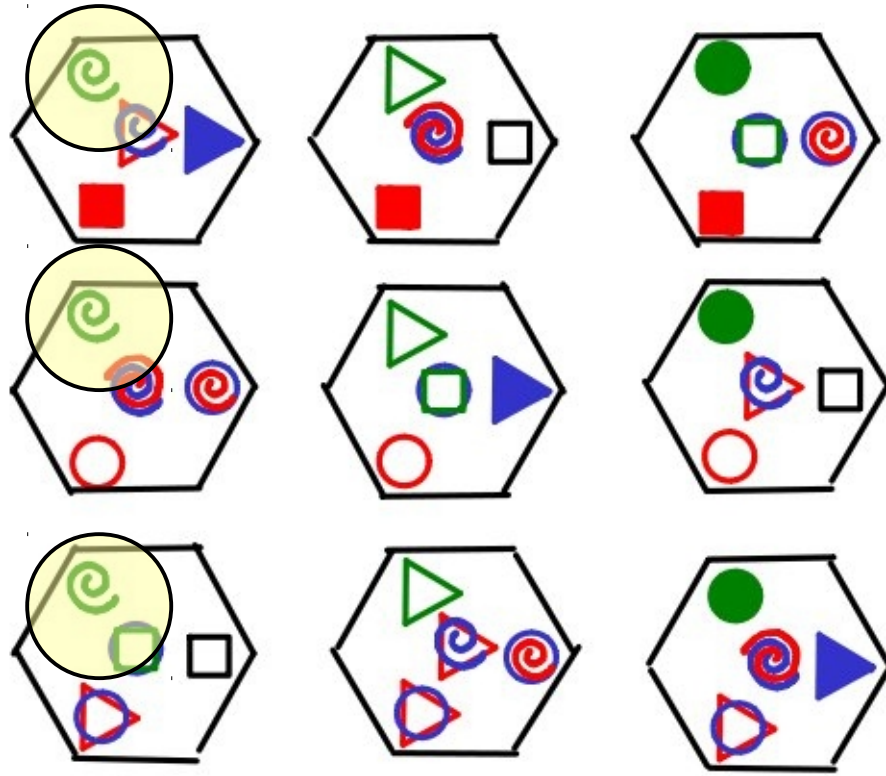
# Classroom Activity

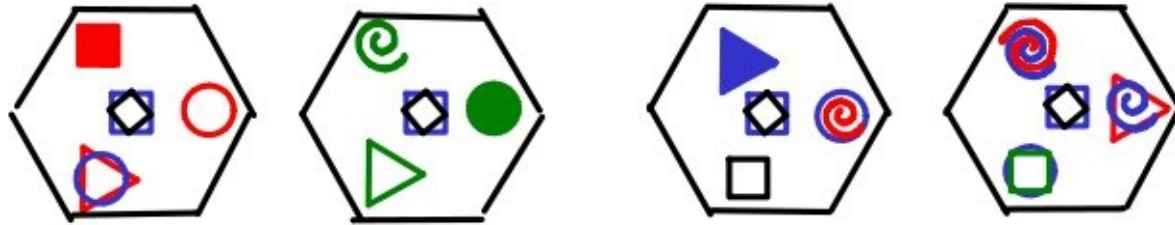
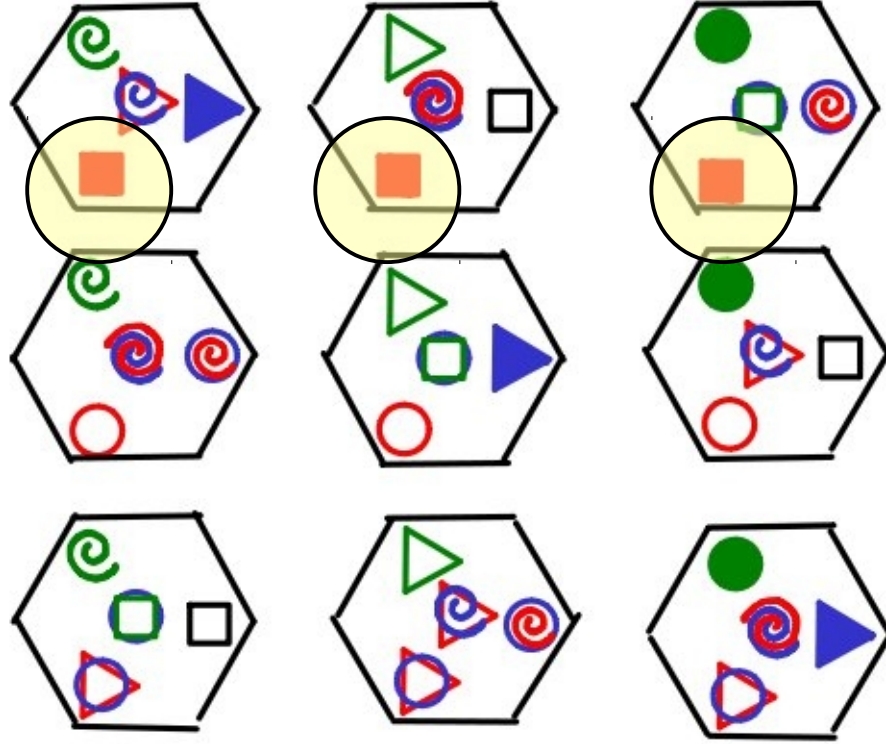




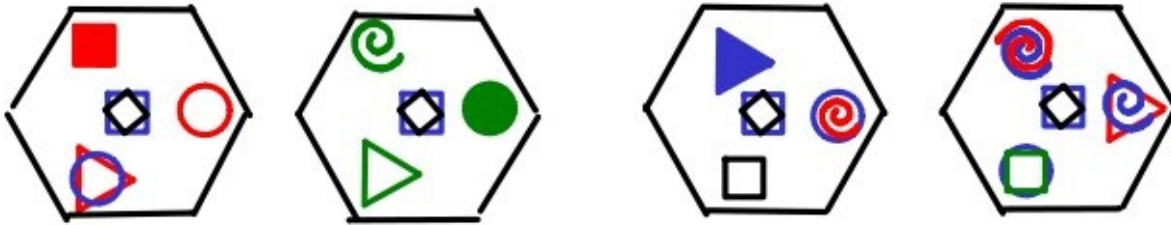
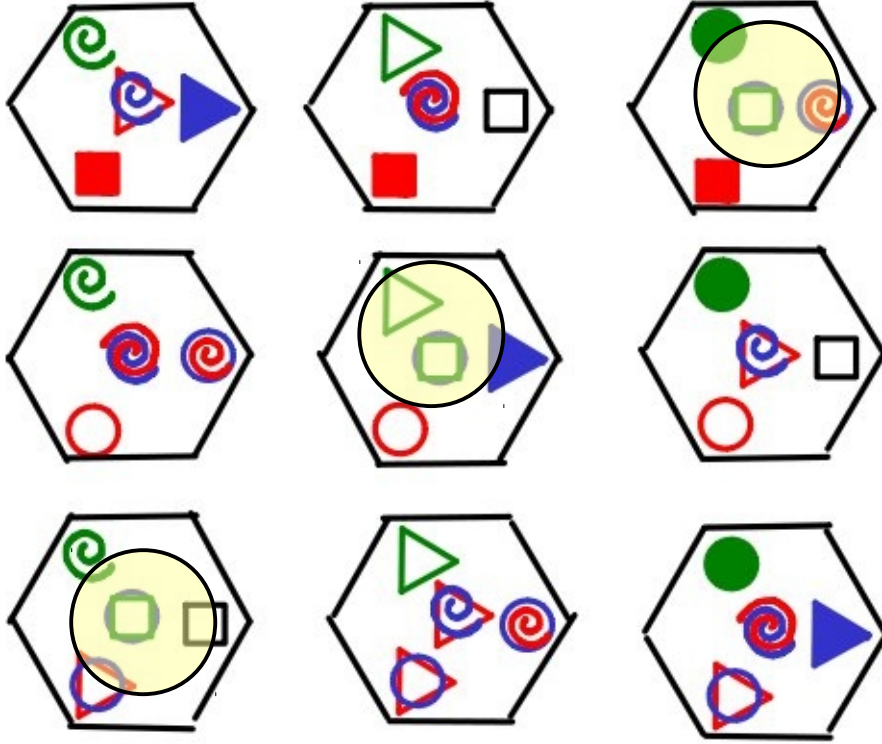


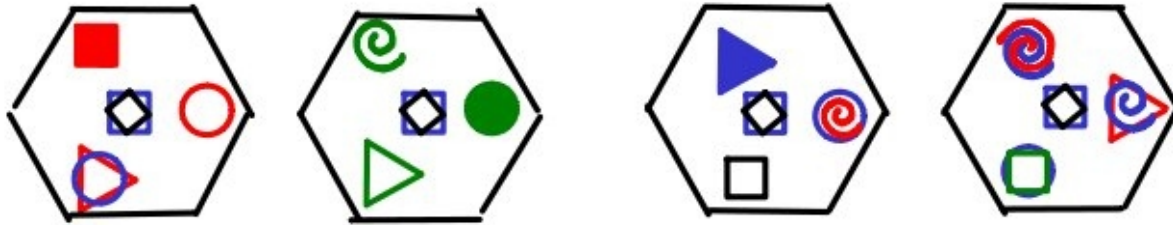
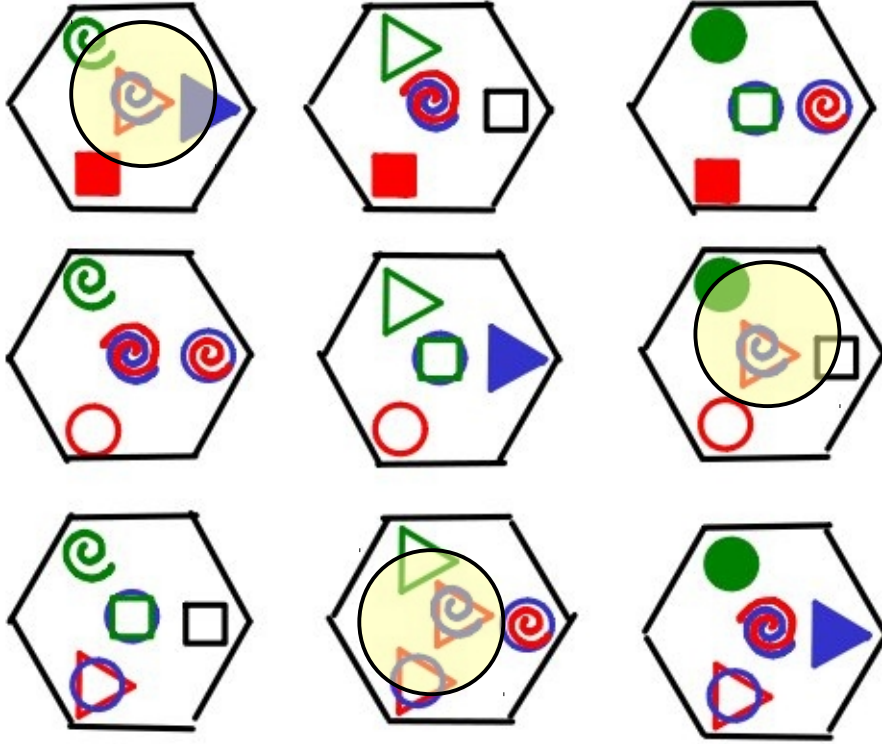


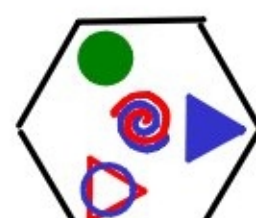
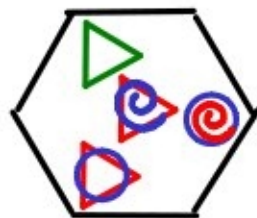
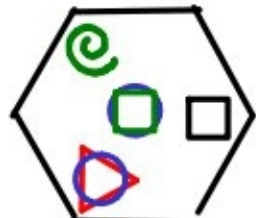
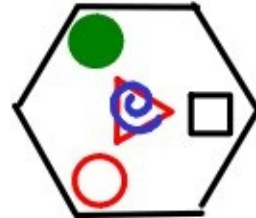
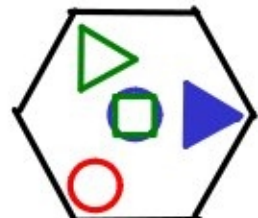
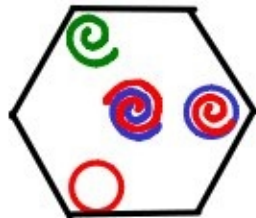
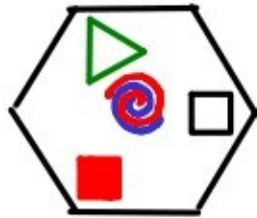
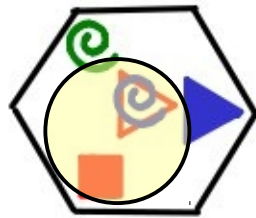
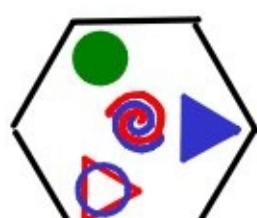
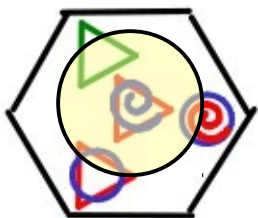
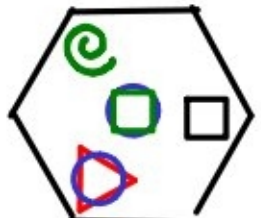
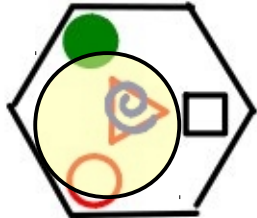
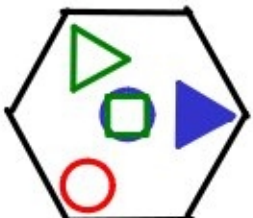
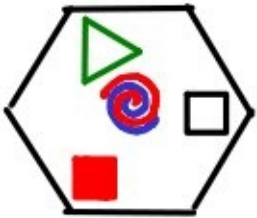
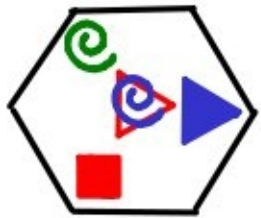




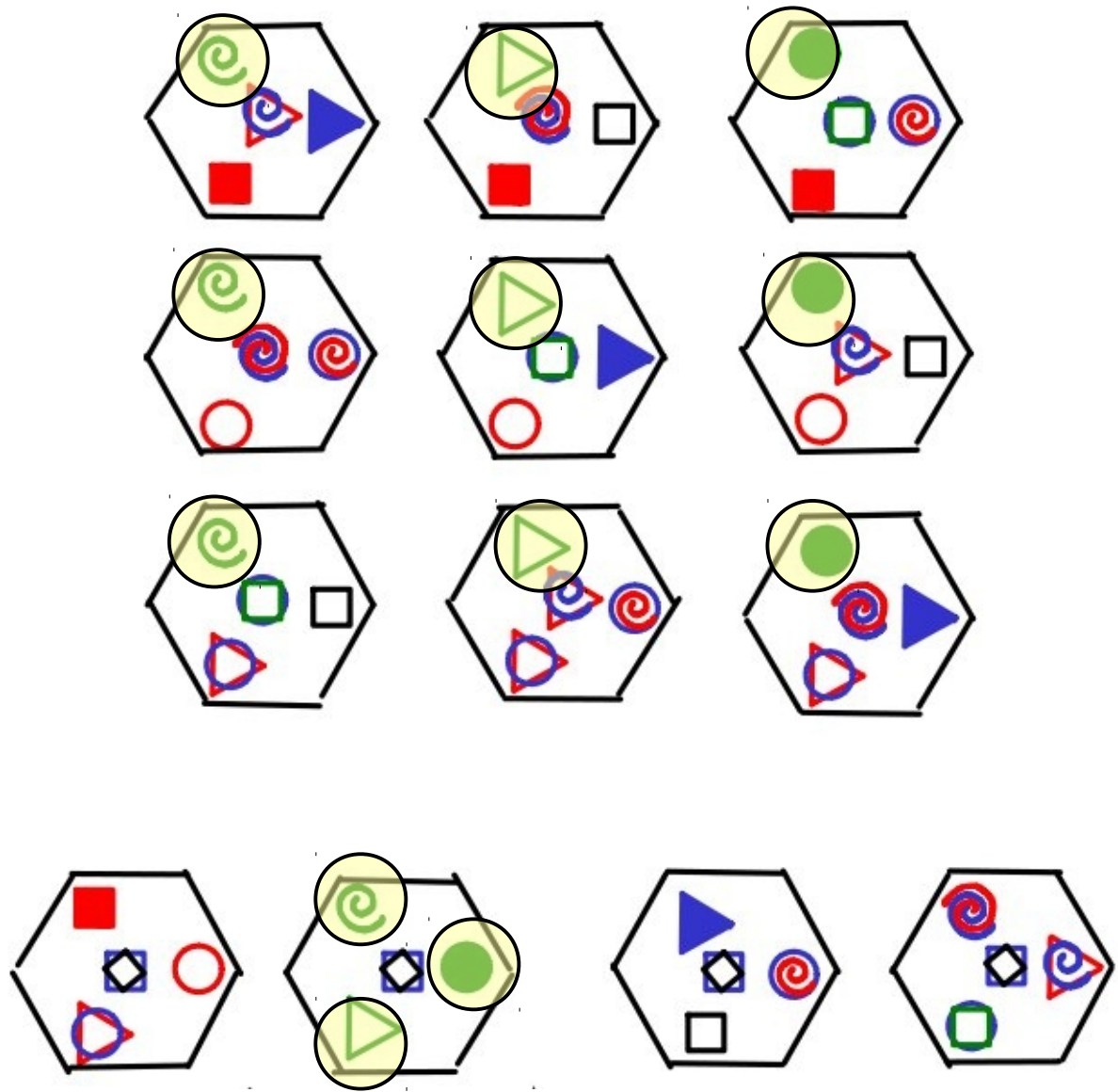




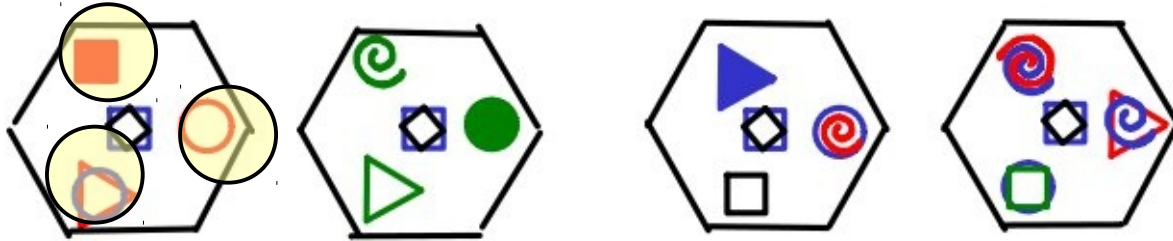
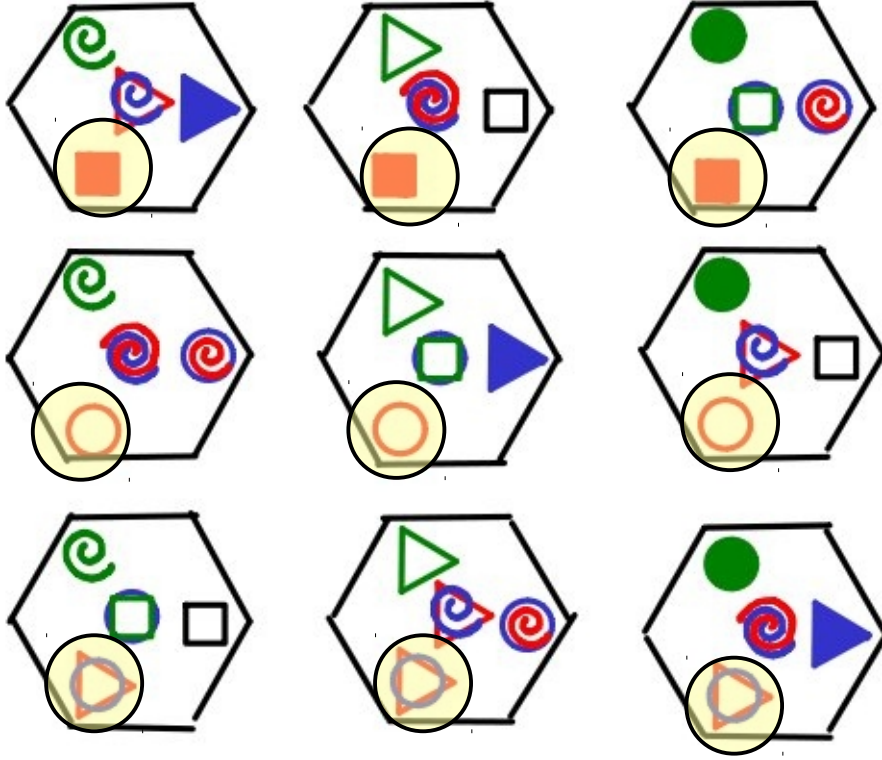


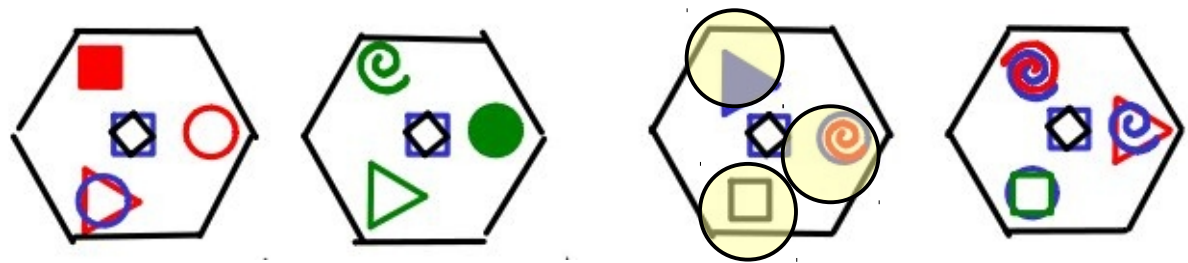
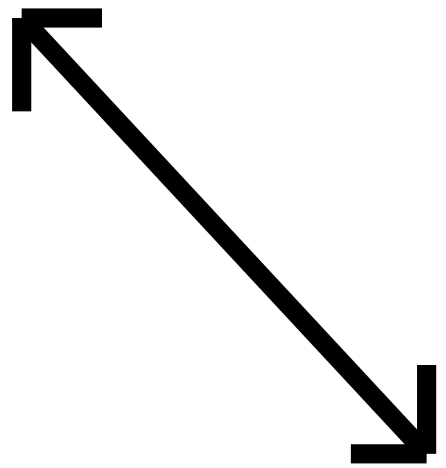
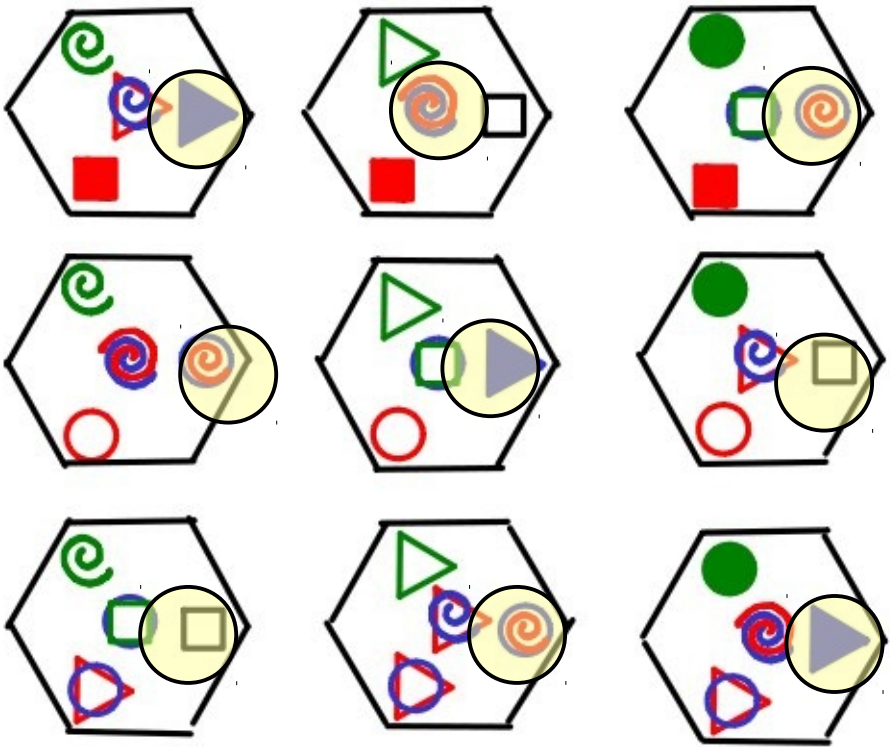








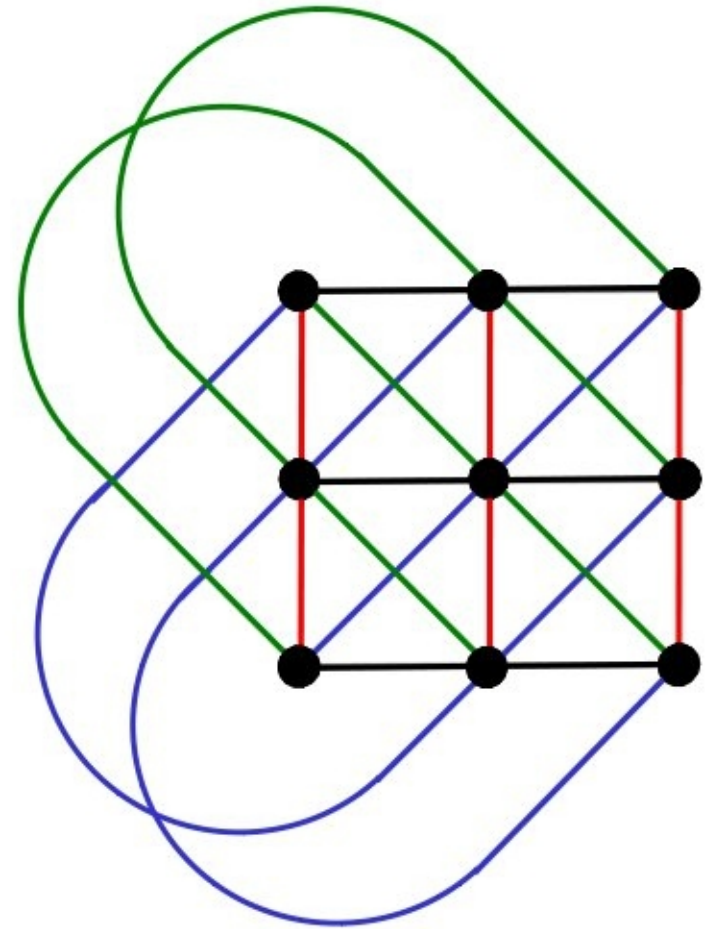
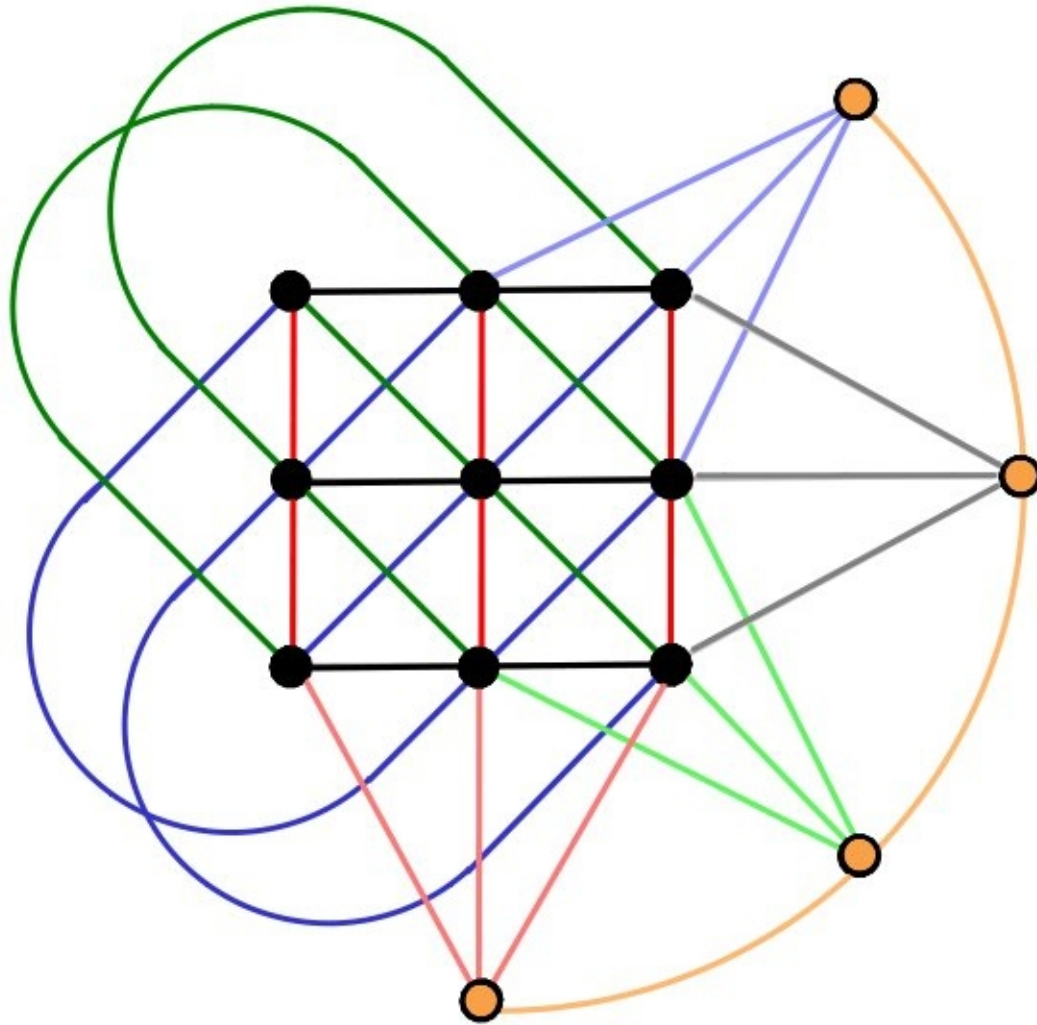








# $n=3$ Projective Plane & Affine Plane



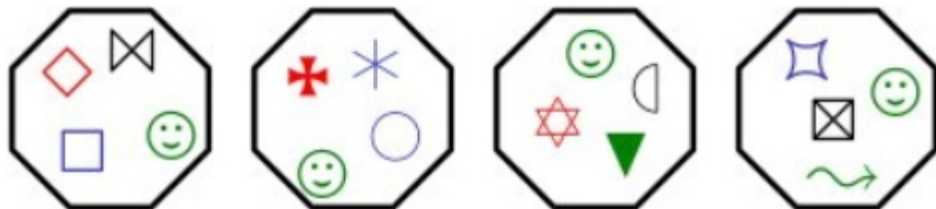
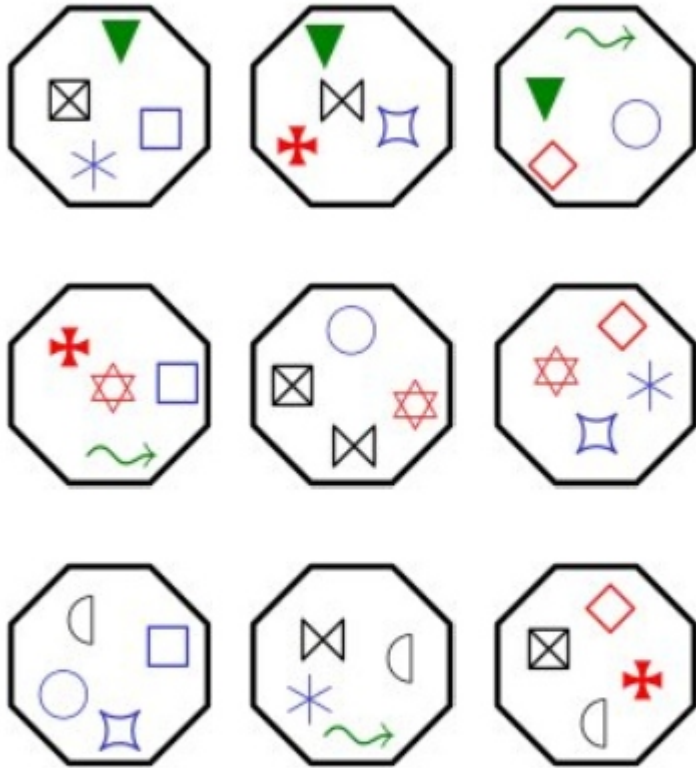
# Biggest Challenges:

- Problem is too small to properly assess understanding in students; they appear to understand it better than they do.
- Explaining the torus/wrapping for diagonals.
- Convincing them that there are two families of diagonals, rather than just one.
- They need feedback while studying.
  - This is why I wrote the Java/Javascript apps.

# <http://www.donnadietz.com>

The screenshot shows a Mozilla Firefox browser window with the address bar displaying [www.donnadietz.com/projective3.html](http://www.donnadietz.com/projective3.html). The game interface includes a 'New Game' button and a grid of 13 octagonal tiles. The tiles are arranged in a 3x3 grid with a fourth tile below the bottom-right corner. Each tile contains a unique combination of symbols from a finite projective geometry. The symbols include: a blue square, a blue circle, a blue four-pointed star, a blue crescent moon, a blue square with an 'X', a blue six-pointed star, a blue triangle, a green triangle, a green wavy line, a green smiley face, a red diamond, a red six-pointed star, a red cross, a red crescent moon, a red square, and a red triangle.

Your goal is to arrange the tiles into a pattern which demonstrates the fundamental properties of a finite projective geometry! (Click and drag a tile on top of an existing tile, and those two tiles will swap.) First, pull aside four tiles which have a symbol in common and place them in the rightmost column. Then, arrange the remaining 9 into a square so that each row, each column, and each diagonal has a common symbol. You are on a torus. (That is, a "pac-man" board.) So, there are more diagonals than you can see immediately. Have fun!



# YouTube explanation

Math154FinalReviewQ3

3) For the Finite Projective Plane given, lightly cross out a collection of cards corresponding to a symbol of your choice. Then, redraw the remaining cards below in a square grid so that each row has a common symbol, each column has a common symbol, and each diagonal and counterdiagonal has a common symbol.

*will become Affine Plans*

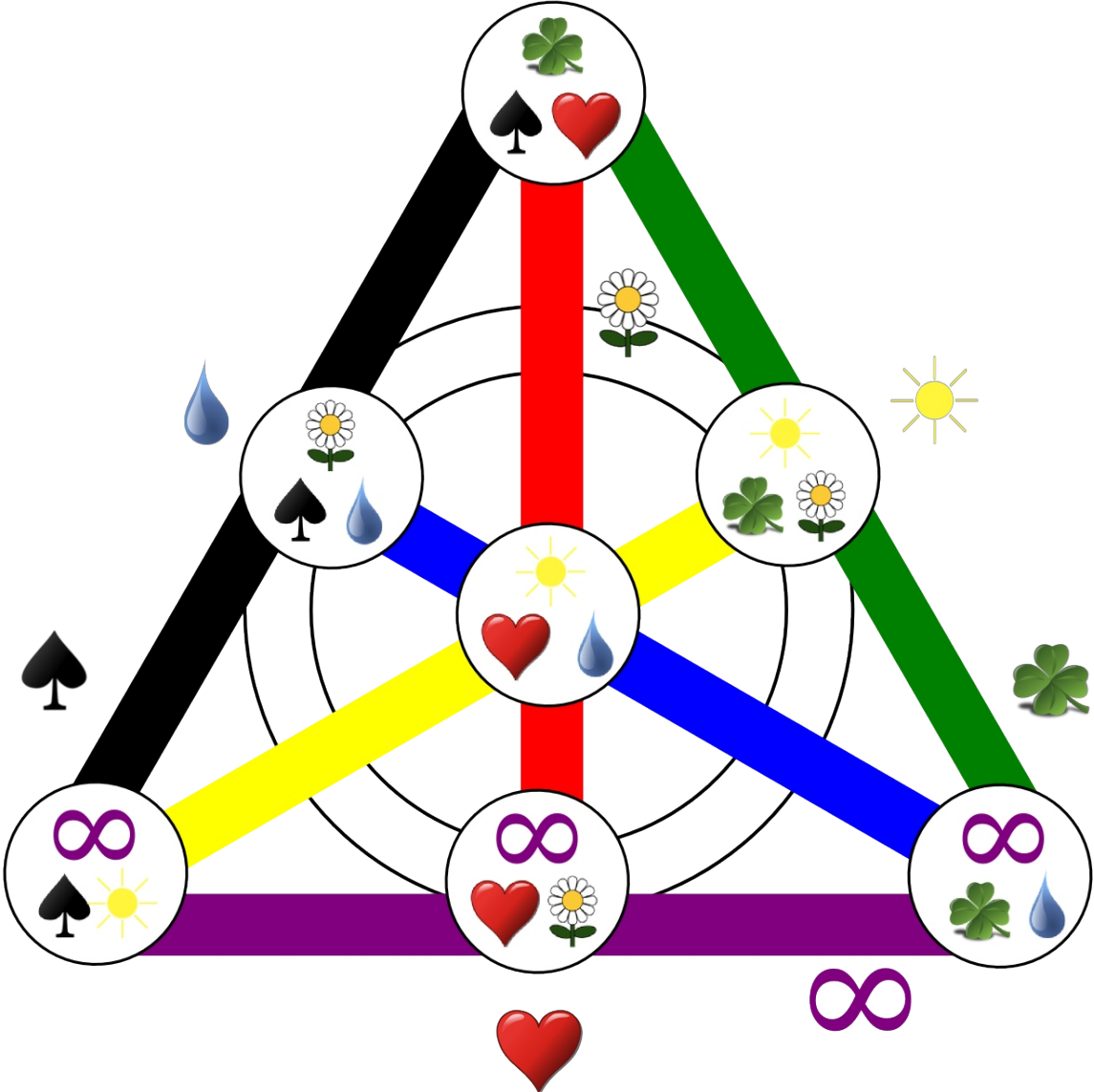

*V* *H*

0:00 / 7:20

YouTube



Maxime Bourrigan, *Dobble et la geometrie finie*.  
Images de Mathematiques, CNRS 2011





The Fano Plane is the  $n=2$   
Finite Projective Plane

Children's Spot-it! is the  $n=5$   
Finite Projective Plane

Spot-it! (Regular) is the  $n=7$   
Finite Projective Plane





# Main Points:

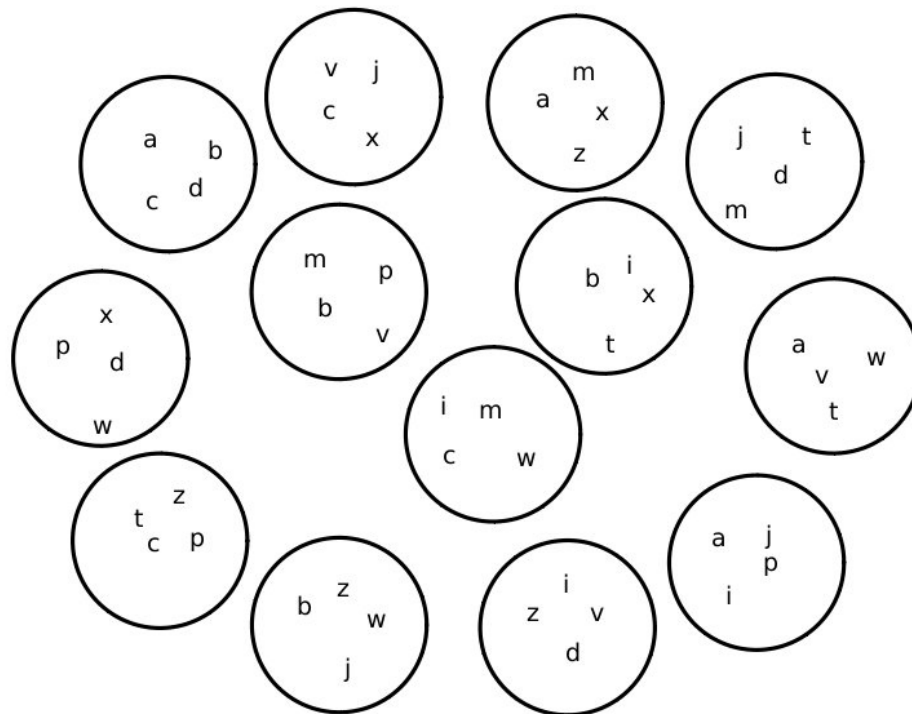
- $n$  is prime
- Any arbitrary pair of cards has exactly one common symbol.
- Any arbitrary pair of symbols is present on exactly one card.

For  $n=3$ :

Exam Questions

# Test Questions:

Lightly cross out four cards. Redraw the remaining cards in a 3x3 grid representing an affine plane.





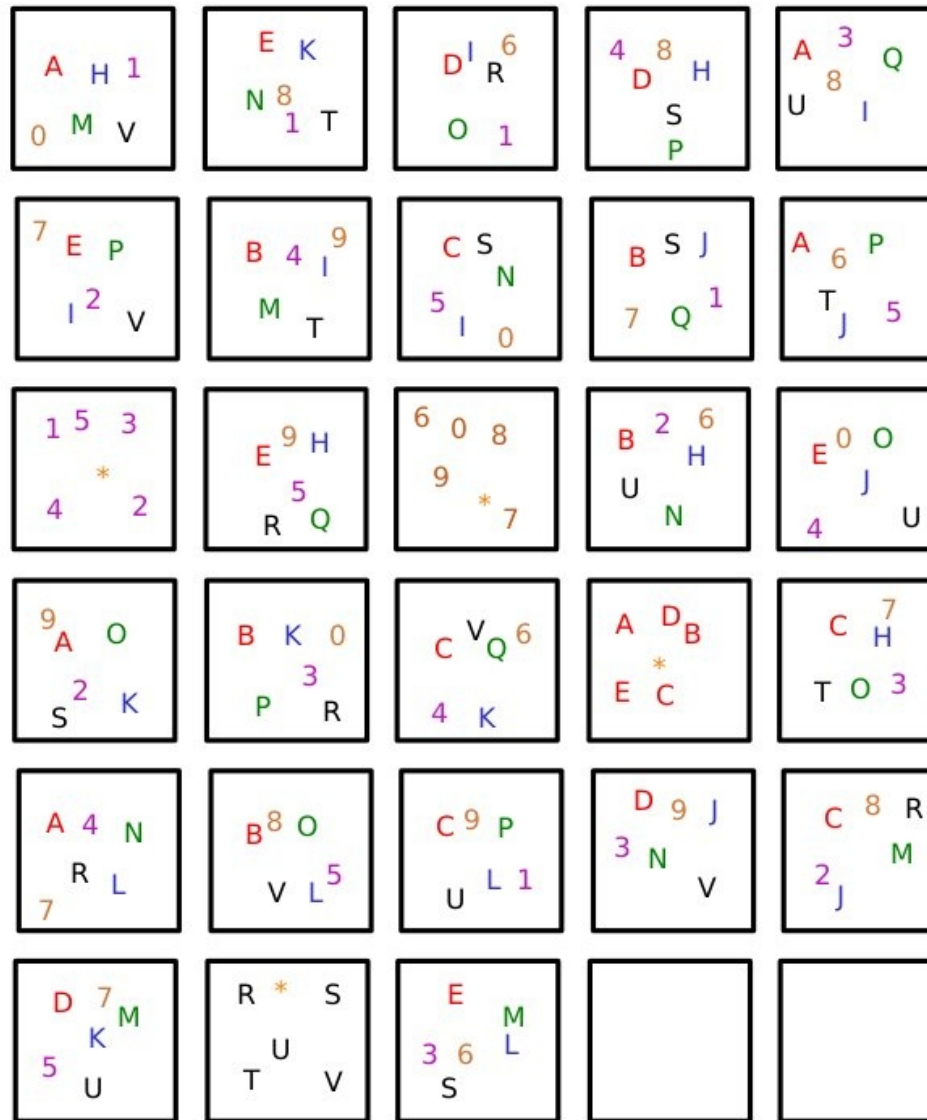
# Results:

- Great problem! They liked it!
- You need to do one like in in class.
- It suffices to set rows/columns (diagonals free).
- Biggest problem: convincing students to use the cards you have provided. (They prefer to just draw their own new set of cards, which would be a good, but different, problem.)

For  $n=5$ :

Classroom Activities  
Javascript App (which I wrote)  
Exam Questions

**Classroom Activity:  
Find the missing cards**



What is missing? (Three cards.)

They contain these symbols:

\*\* D H I J K L L M N O P Q Q T 2 0

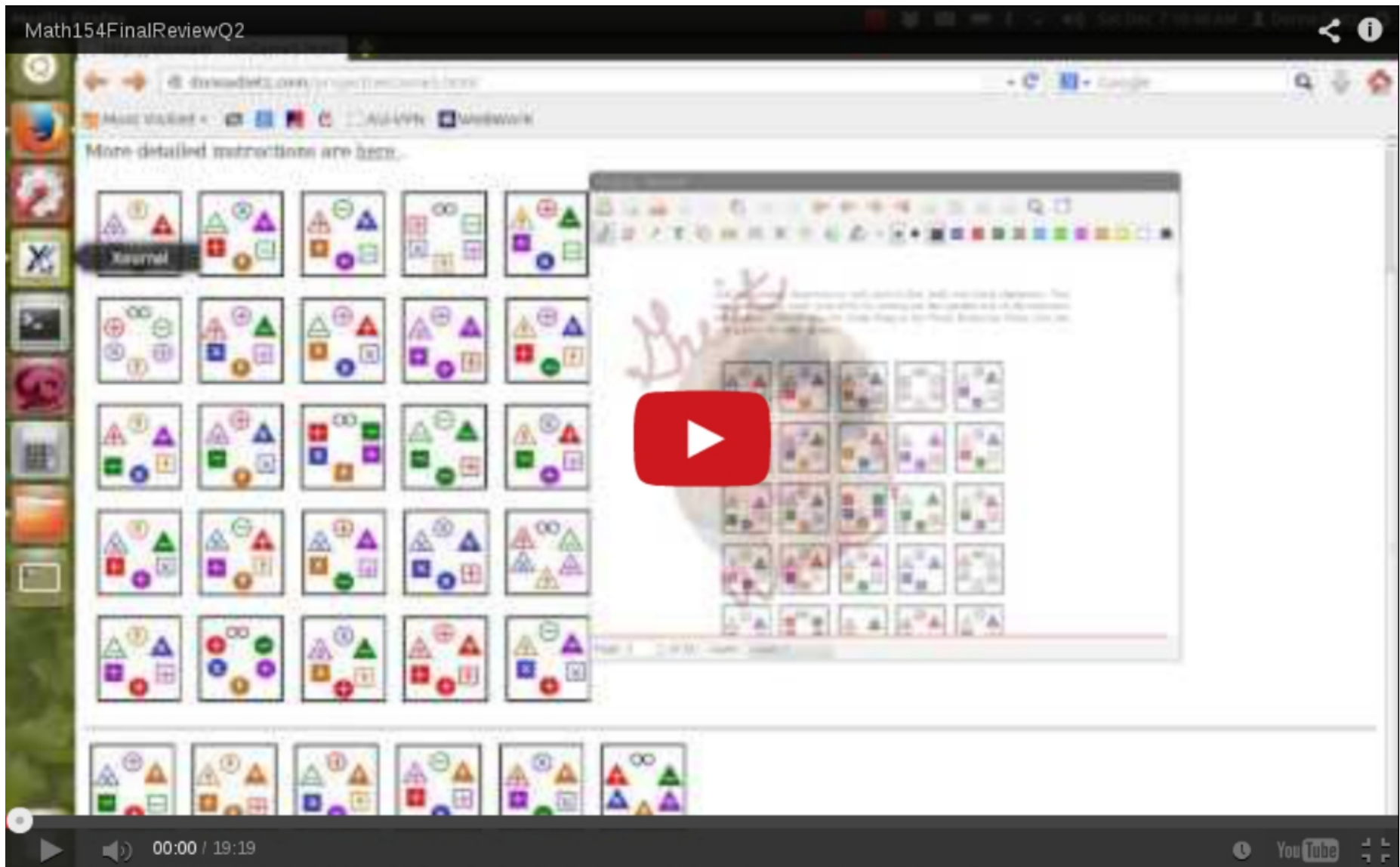




# Results:

- Good for group-work
- Still struggling with the two diagonal families not being the same, and now the  $m=2$  ,  $m=-2$  families also.
- Drives home fact that each pair of symbols must occur together exactly one time, and each pair of cards contains common symbol.
- $n=5$  Projective Plane complex enough to learn main concepts.
- Lends itself well to hints along the way.
  - “Which symbols are missing twice?”
  - “What if one symbol were missing three times?”

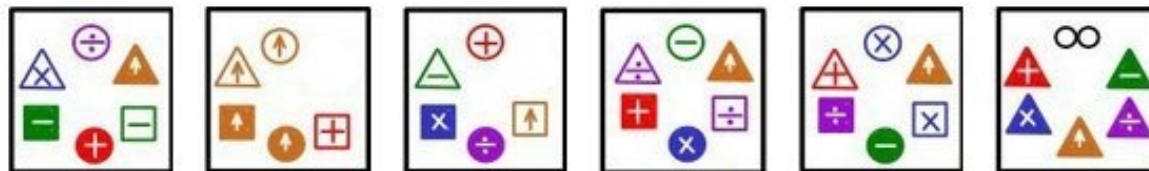
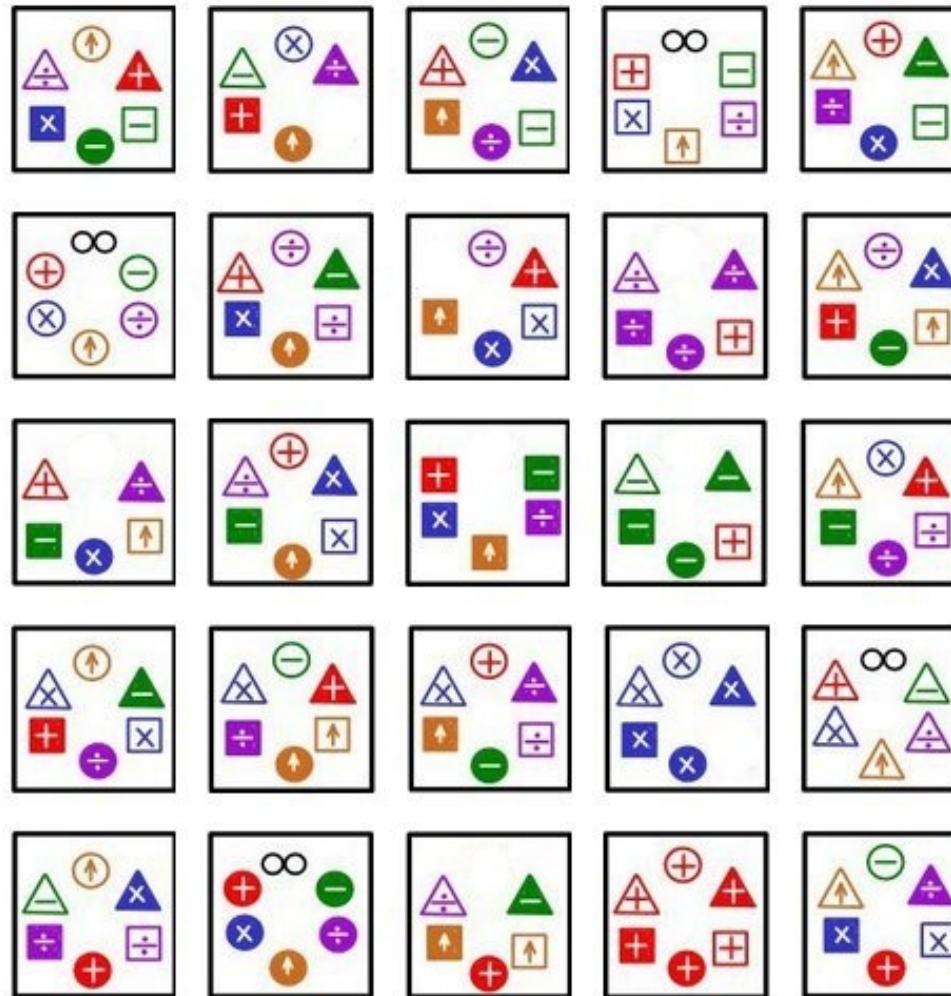
# Javascript app and YouTube help



# n=5 Exam Questions

- Sometimes given Affine Plane only
- Find missing symbols
- Find parallel families
- Find missing cards

# Fill-in missing symbols:



# Fill-in symbols and give parallel sets:




# Results:

- Regular alignment of artwork is best, so missing symbols are obvious.
- Randomly removed symbols are easier to replace than entire lines.
- Students prefer having the infinity line present.

# And Much Much More

15 minutes is not enough.

Visit [http:// www. donnadietz. com](http://www.donnadietz.com)

Email me

# Student responses:

- My students seemed to enjoy this unit.
- Anytime the scissors or glue come out, their math anxiety seems to drop and they go into fun mode. (Art can't be math, right?)
- My students nominated me for the CTRL Jack Child CTRL Teaching with Technology Award because I wrote these apps, so I guess that means they liked them. (And I even won the award.)



Center for Teaching, Research & Learning

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