## AR 212

## Win, Lose, or Draw

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\text { TEST } 1
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## Developed by Dr. Karen Holmes

## Group \#1



The Games We Play


1) In your group, list as many reasons as you can think of why people play games.
2) In your group, list all of the different types of games you can (i.e. board games, etc) and list one example of each.
3) List three of your favorite games:
a) played as a child
b) play now
4) 
5) 
6) 
7) 
8) 

## 3)

4) Go to the internet and look up the history of one of those games from both a) and b) above. Take some notes below about when they were created, by whom, and 2 other interesting facts about each game.
a) game:
when created:
who created it:
other interesting facts: 1 .
2. 

b) game:
when created:
who created it:
other interesting facts: 1 .
2.

|  |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  | Paper

Games


2.1) The origin of Tic Tac Toe is not known precisely, but some say it was played in Egypt as early as 1300 BC. In modern history it was actually the first computer game in the world. Play enough games of Tic Tac Toe with the people in your group to fill in everyone's boards. While playing the games, try to come up with strategies of how to win or, at least, avoid losing.
a) If you go first, where should you place your mark?
b) Should you always win if you start?

| corner | side | corner |
| :---: | :---: | :---: |
| side | center | side |
| corner | side | corner |

2.2) Another three-in-a-row game from Zimbabwe is called Tsoro Yemtatu (tatu means "three"). On the diagram below, note that there are 7 intersection points; the object of the game is to get three of your "stones" in a row of intersection points. Each player has three stones which are laid in turn on an intersection point not already occupied. After all stones are laid down, there will be only one open point. The players take turns moving any of their stones to the open point. Whoever gets three in a row wins.


Write any ideas or strategies of how to succeed in winning or avoid losing.
2.3) Another three in a row game played by Asante schoolchildren in southern Ghana is played on a square network marked on the ground. Each player has four sticks or colored stones. Each player in turn places their stones on an intersection point. Once all are placed players may move one of their markers to an adjacent point. The object is to place three in a row.


Write any ideas or strategies of how to succeed or avoid losing.
2.4) If you've ever played the game Mastermind, Bulls and Cows is the paper and pencil version and has been around for about a century. It is a game played with 2 players (or possibly teams). Here are the general rules:


1) Player 1 writes a 4 (or 3-6) digit secret code where all of the digits are unique (i.e. no repeating digits, like 4426).
2) Player 2 makes a guess.
3) Player 1 will say how many bulls and cows the guess contains.

> BULL $=a$ correct digit in the right position
> COW $=a$ correct digit in the wrong position
4) Player 2 keeps making guesses until they figure out the secret code.

Example Game:
Player 1's Secret Code: 4271
Player 2's Guess \#1: 1234
Player 2’s Guess \#2: 1567
Player 1 would say: 1 Bull (for the 2 ) and 2 Cows (for the 1 and 4)
Player 2's Guess \#3: 1890
Player 1 would say: 0 Bulls and 2 Cows (for the 1 and 7)
Player 1 would say: 0 Bulls and 1 Cow (for the 1 )
Now Player 2 knows 1 is in the code, but 8,9 , and 0 are not, so they keep guessing....

Play a few games of Bulls and Cows with your group, either breaking into pairs and playing one on one or play two on two. Keep track of how many guesses it takes each of you to get the right code.
a) Using a $\mathbf{3}$ digit code:

Game 1: code $\qquad$ \# of guesses $\qquad$ Game 2: code $\qquad$ \# of guesses $\qquad$

Game 3: code $\qquad$ \# of guesses $\qquad$ Game 4: code $\qquad$ \# of guesses $\qquad$
b) Were there any strategies that you used to make you get the code quicker?
c) Using a 4 digit code:

Game 1: code $\qquad$ \# of guesses $\qquad$ Game 2: code $\qquad$ \# of guesses $\qquad$
d) What differences did you observe when using a 4 digit code as opposed to a 3 digit code?
e) Now try a 6 digit code:

Game 1: code $\qquad$ \# of guesses $\qquad$ Game 2: code $\qquad$ \# of guesses $\qquad$
2.5) In the game Bulls and Cows, a BULL = correct digit in the right position and a COW = correct digit in the wrong position. Determine the code for each example.
a) Guess 1:123 $\rightarrow \quad 1$ bull and 0 cows Guess 2: $456 \quad \rightarrow \quad 0$ bulls and 1 cow Guess 3: 789 $\rightarrow \quad 0$ bulls and 1 cow

Code: $\qquad$ Guess 4: $147 \quad \rightarrow \quad 1$ bull and 2 cows
b) Guess 1:123 $\quad \rightarrow \quad 1$ bull and 0 cows
Guess 2: $456 \quad \rightarrow \quad 0$ bull and 1 cow

Guess 3: 789 $\rightarrow \quad 1$ bull and 0 cows
Guess 4: $714 \quad \rightarrow \quad 0$ bulls and 0 cows
Guess 5: $385 \quad \rightarrow \quad 0$ bulls and 0 cows
Code: $\qquad$
c) Guess 1: $123 \quad \rightarrow \quad 0$ bulls and 1 cow

Guess 2: $456 \quad \rightarrow \quad 1$ bull and 0 cows
Guess 3: $789 \quad \rightarrow \quad 0$ bulls and 0 cows
Guess 4: $410 \quad \rightarrow \quad 0$ bulls and 1 cow
Guess 5: $206 \rightarrow 0$ bulls and 2 cows
Code: $\qquad$
2.6) During a crazy weekend of paintball, four friends were having great fun. The paint came in blue, green, yellow and red. Coincidentally, the four friends had T-shirts in those same colors. Figure out what color paint and t-shirt they each had using the following clues:

Brenda used blue paint balls.
The person in the green T-shirt used yellow paint balls.
James was not wearing a red T-shirt.
Diane used green paint balls and wore a blue T-shirt.
Simon was the only person who used paint which was the same color as his T-shirt.

|  | Brenda | James | Diane | Simon |
| :---: | :---: | :---: | :---: | :---: |
| Paint |  |  |  |  |
| T-shirt |  |  |  |  |

2.7) During a recent inter-school athletic event, four girls competed in the 10,000 meter race. The girls were named: Jane, Lucy, Anne, and Mimi and their surnames were: Guest, Brown, Joseph, and Scott. Figure out in what order they finished the race using the following clues:

Jane Brown beat Lucy.
Miss Guest beat Mimi.
Anne was not third.
Miss Joseph was not last.
Miss Scott, who was not Anne, came in immediately after Jane.

| Place | $1^{\text {st }}$ | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ |
| :---: | :---: | :---: | :---: | :---: |
| First name |  |  |  |  |
| Last name |  |  |  |  |

## GRE / LSAT Practice

If you plan on going to grad school or law school, you will probably be asked to take the GRE (Graduate Record Exam) or LSAT (Law School Admission Test). It aims to measure verbal, quantitative, and analytical skills that have developed throughout a person's life. Here are a few of questions similar to some on the GRE or LSAT.
3.1) A nonprofit organization's board of directors, composed of four women (Angela, Betty, Carmine, and Delores) and three men (Ed, Frank, and Grant), holds frequent meetings. A meeting can be held at Betty's house, at Delores's house, or at Frank's house. The following conditions must hold:


Delores cannot attend any meetings at Betty's house.
Carmine cannot attend any meetings on Tuesday or on Friday.
Angela cannot attend any meetings at Delores's house.
Ed can attend only those meetings that Grant also attends.
Frank can attend only those meetings that both Angela and Carmine attend.
a) If all members of the board are to attend a meeting, under which of the following circumstances can it be held?
i) Monday at Betty's
iv) Thursday at Frank's
ii) Tuesday at Frank's
v) Friday at Betty's
iii) Wednesday at Delores's
b) Which of the following can be the group that attends a meeting on Wednesday at Betty's?
i) Angela, Betty, Carmine, Ed, and Frank
iv) Angela, Betty, Delores, Frank, and Grant
ii) Angela, Betty, Ed, Frank, and Grant
v) Angela, Betty, Carmine, Frank and Grant
iii) Angela, Betty, Carmine, Delores, and Ed
c) If Carmine and Angela attend a meeting but Grant is unable to attend, which of the following could be true?

The meeting is: i) on Tuesday
iv) at Frank's
ii) on Friday , v) attended by six board members
iii) at Delores's
d) If the meeting is held on Tuesday at Betty's, which of the following pairs can be among the members who attend?
i) Angela and Frank
iv) Ed and Betty
ii) Carmine and Ed
v) Frank and Delores
iii) Carmine and Angela
e) If Frank attends a meeting on Thursday that is not held at his house, which of the following must be true?
i) The group can include, at most, two women.
iv) Grant is not at the meeting.
ii) The meeting is at Betty's house.
v) Delores is at the meeting.
iii) Ed is not at the meeting.
f) If Grant is unable to attend a meeting on Tuesday at Delores's, what is the largest possible number of members who can attend?
g) If a meeting is held on Friday, list the board member(s) who cannot attend.

3.2) An amusement park roller coaster includes five cars, numbered 1 through 5 from front to back. Each car accommodates up to two riders, seated side by side. Six people-Tom, Gwen, Laurie, Mark, Paul and Jack - are riding the coaster at the same time.

Laurie is sharing a car.
Mark is not sharing a car and is seated immediately behind an empty car.
Tom is not sharing a car with either Gwen or Paul. Gwen is riding in either the third or fourth car.
a) Which of the following groups of riders could occupy the second car?
(A) Laurie only
(D) Jack and Tom
(B) Tom and Gwen
(E) Jack, Gwen, and Paul
(C) Laurie and Mark
b) If Gwen is riding right behind Laurie's car and right ahead of Tom's car, all of the following must be true EXCEPT:
(A) Gwen is riding in the fourth car.
(D) Laurie is riding in the third car.
(B) Paul is riding in the third car.
(E) The first car is empty.
(C) Tom is riding in the fifth car.
c) Which one of the following statements CANNOT be true?
(A) Neither Tom nor Gwen is sharing a car with another rider.
(B) Neither Mark nor Jack is sharing a car with another rider.
(C) Tom is sharing a car, and Jack is sharing a car.
(D) Gwen is sharing a car, and Paul is sharing a car.
(E) Tom is sharing a car, and Gwen is sharing a car.
d) If Paul is riding in the second car, how many different combinations of riders are possible for the third car?
(A) one
(B) two
(C) three
(D) four
(E) five
e) Assume that a seventh rider is riding with Jack in the first car, but that all other rules remain unchanged. Which of the following is a complete and accurate list of the riders who might be riding in the fifth car?
(A) Mark
(D) Tom, Laurie, Mark
(B) Gwen, Paul
(E) Mark, Gwen, Paul, Tom, Laurie
(C) Tom, Laurie, Paul
3.3) A baseball league has six teams: A, B, C, D, E, and F. All games are played at 7:30pm on Fridays, and there are sufficient fields for each team to play a game every Friday night. Each team must play each other team exactly once, and the following conditions must be met:

Team A plays team D first and team F second.
Team B plays team E first and team C third.
a) What is the total number of games that each team must play during the season?
b) What team must team B play second?
c) The last set of games could be between which teams?
i) A and B; C and F; D and E
iv) A and D; B and C; E and F
ii) A and B; C and D; E and F
v) A and D ; B and E ; C and F
iii) $A$ and $C$; $B$ and $E$; D and $F$
d) If team D wins five games, which of the following must be true?
i) Team A loses five games.
iv) Team $B$ wins five games.
ii) Team A wins four games.
v) Team B loses at least one game.
iii) Team A wins its first game.

3.4) In an executive parking lot, there are six parking spaces in a row; labeled 1 through 6 . Exactly five cars of 5 different colors- black, gray, pink, white, and yellow- are to be parked in the spaces. The cars can park in any of the spaces as long as the following conditions are met:

The pink car must be parked in space 3 .
The black car must be parked in a space next to the space in which the yellow car is parked. The gray car cannot be parked in a space next to the space in which the white car is parked.
a) If the yellow car is parked in space 1 , how many acceptable parking arrangements are there for the five cars?
b) Which of the following must be true of any acceptable parking arrangement?
i) One of the cars is parked in space 2.
ii) One of the cars is parked in space 6.
iii) There is an empty space next to the space in which the yellow car is parked.
iv) There is an empty space next to the space in which the gray car is parked.
v) Either the black car or the yellow car is parked in a space next to space 3 .
c) If the gray car is parked in space 2 , none of the cars can be parked in which space?
d) What spaces can the white car not park in?
3.5) Two collectors, John and Jack, are each selecting a group of three posters from a group of seven movie posters: J, K, L, M, N, O, and P. No poster can be in both groups. The selections made by John and Jack are subject to the following restrictions:

If K is in John's group, M must be in Jack's group.
If N is in John's group, P must be in Jack's group.
J and P cannot be in the same group.
M and O cannot be in the same group.

a) Which of the following pairs of groups selected by John and Jack conform to the restrictions?

| i) John: J, K, L | Jack: | M, N, O |
| :--- | ---: | :--- |
| ii) | J, K, P | L, M, N |
| iii) | K, N, P | J, M, O |
| iv) | L, M, N | K, O, P |
| v) | M, O, P | J, K, N |

b) If N is in John's group, which poster(s) could not be in Jack's group?
c) If K and N are in John's group, what three posters must be in Jack's group?
d) If J is in Jack's group, which of the following is true?
i) K cannot be in John's group
iv) P must be John's group
ii) N cannot be in John's group
iii) O cannot be in Jack's group
e) If K is in John's group, which of the following is true?
i) J must be in John's group
ii) O must be in John's group
iii) L must be in Jack's group
v) P must be in Jack's group
iv) N cannot be in John's group
v) O cannot be in Jack's group
3.6) An athlete has six trophies to place on an empty three-shelf display case. The six trophies are bowling trophies $\mathrm{F}, \mathrm{G}$, and H and tennis trophies $\mathrm{J}, \mathrm{K}$, and L . The three shelves of the display case are labeled 1 to 3 from top to bottom. Any of the shelves can remain empty and hold any number of trophies. The athlete's placement of the trophies must conform to the following conditions:

- J and L cannot be of the same shelf.
- F must be on the shelf immediately above the shelf that L is on.
- No single shelf can hold all three bowling trophies.
- K cannot be on Shelf 2 .
a) If G and H are on Shelf 2, which of the following must be true?
A) K is on Shelf 1 .
D) G and J are on the same shelf.
B) $L$ is on Shelf 2.
E) F and K are on the same shelf.
C) $J$ is on Shelf 3 .
b) If no tennis trophies are on Shelf 3, which pair of trophies must be on the same shelf?
A) $F$ and $G$
D) K and J
B) L and H
E) G and H
C) L and G
c) If J is on Shelf 2, which of the following must also be on Shelf 2?
A) K
B) $G$
C) F
D) L
E) H
d) If Shelf 1 remains empty, which of the following must be FALSE?
A) H and F are on the same shelf.
B) There are exactly three trophies on Shelf 2.
C) G and H are on the same shelf.
D) There are exactly two trophies on Shelf 3 .
E) G and K are on the same shelf.
e) If $L$ and $G$ are on the same shelf, and if one of the shelves remains empty, which of the following must be true?
A) If $H$ is on Shelf 3 , then $J$ is on Shelf 2.
B) K and L are on the same shelf.
C) If $H$ is on Shelf 2, then J is on Shelf 3.
D) F and K are on the same shelf.
E) If J is on Shelf 2, then H is on Shelf 1.
3.7) After the goalie has been chosen, the Smalltown Bluebirds hockey team has a starting lineup that is selected from 2 groups. $1^{\text {st }}$ Group: John, Dexter, Bart, Erwin and $2^{\text {nd }}$ Group: Leanne, Roger, George, Marlene, Patricia. When deciding on the players in the lineup, the coach considers the following requirements:

George will only start if Bart also starts.


Dexter and Bart will not start together.
If George starts, Marlene won't start.
The 4 fastest players are: John, Bart, George and Patricia
3 of the 4 fastest players will always be chosen.
Two players are always chosen from the first group, while three are chosen from the second group.
a) If Marlene is in the starting lineup, which of the players will be the first group players who will also be starting?
A. Erwin and Dexter
C. John and Bart
E. Erwin and Bart
B. John and Dexter
D. Dexter and Bart
b) Of the following hockey players, who must start?
A. Patricia
C. George
E. Marlene
B. John
D. Bart


## Logic Puzzles


4.1) Five people are tennis players. From the clues below, work out the club where each plays (Worthies, Servers, Nets, Matches, Racquets), what their surname is (Atkins, Evans, Harrison, Kelly, Osuna), and how old they are (18, 30, 43, 55, 61)?

1. Margaret joined Worthies club last summer, and hopes to still be playing when she hits 40 .
2. Stephen isn't at Servers, whose member is the second oldest and isn't a Kelly.
3. Anne, the youngest player, isn't a Harrison, and doesn't play at Nets, the club of Mr Osuna.
4. Racquets only has male members - Carlos would never join it and Margaret Kelly can't!
5. Clive is the second eldest player, being junior to Stephen Atkins.

| First Name | Anne | Carlos | Clive | Margaret | Stephen |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Surname |  |  |  |  |  |
| Club |  |  |  |  |  |
| Age |  |  |  |  |  |

4.2) In horse racing, there are three races that make up the Triple Crown: the Kentucky Derby, the Preakness, and the Belmont. All three races have only been won by the same horse nine times in over 100 years of events. Let's say there are four popular horses signed up to run in the Kentucky Derby (Galloping Grocer, Alec's Pride, OneForTheMoney, EatMyDust). While all four of them placed in the top four winners, none of them placed in the same position as their starting post position (the gate number where each horse started the race from: either 2, 5, 6, 7). Determine the name of each racehorse, their starting post positions, the winning placement of each horse, and the stable colors of each racehorse (Green and Black, Hot Pink, Blue and White, Purple and White).

1. Galloping Grocer didn't have hot pink stable colors.
2. The fourth place horse didn't have the post position 2.
3. The second place horse had a larger numbered post position than OneForTheMoney, but a smaller post position than Galloping Grocer.
4. Eat My Dust had a post position one larger than the horse with blue \& white stable colors.
5. The purple and white stable colors belonged to the horse who won third place.
6. The horse with post position 7 won the race.
7. Alec's Pride had the green \& black stable colors.
8. The horse at post position 5 was not OneForTheMoney.

| Place | $1^{\text {st }}$ | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Name |  |  |  |  |
| Post Position |  |  |  |  |
| Stable Colors |  |  |  |  |

4.3) CLUE: Four friends: Bob, Sue, Allen, and Erin, played a very intense game of Clue. A weapon, suspect, and room card were secretly chosen and the rest of the cards were handed out. Bob and Sue each had 5 cards in their hand while Allen and Erin each only had 4. Based on the following information, figure out who had what cards and from that determine who killed Mr. Boddy, with what weapon, and in what room? (No one bluffs unless it's specifically noted.)

Mark $\sqrt{ }=$ don't have the card
= have the card

| SUSPECTS | B | S | A | E |
| :---: | :---: | :---: | :---: | :---: |
| Colonel Mustard |  |  |  |  |
| Professor Plum |  |  |  |  |
| Mr. Green |  |  |  |  |
| Mrs. Peacock |  |  |  |  |
| Miss Scarlett |  |  |  |  |
| Mrs. White |  |  |  |  |
| WEAPONS |  |  |  |  |


| Knife |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- |
| Candlestick |  |  |  |  |
| Revolver |  |  |  |  |
| Rope |  |  |  |  |
| Lead Pipe |  |  |  |  |
| Wrench |  |  |  |  |


| ROOMS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Hall |  |  |  |  |
| Lounge |  |  |  |  |
| Dining Room |  |  |  |  |
| Kitchen |  |  |  |  |
| Ballroom |  |  |  |  |
| Conservatory |  |  |  |  |
| Billiard Room |  |  |  |  |
| Library |  |  |  |  |
| Study |  |  |  |  |

- Neither Erin nor Sue had Miss Scarlet.
- Bob asked Allen if he had Mrs. Peacock, Revolver, and Hall, but Allen had none of these cards.
- Erin asked Allen if he had Prof Plum, Lead Pipe, and Dining Room; Allen had one of these which he showed her.
- Bob had more than one weapon, while Allen just had one.
- Sue asked Erin if she had Mrs. Peacock, Knife, and Billiard Room. Erin has 2 of these and showed Sue the weapon.
- Allen dropped the Lounge card, which was his only room, on the floor and all the other players saw it.
- Erin was upset that she did not get Mrs. Peacock, her favorite character.
- Bob accidentally showed Allen the Conservatory even though that was not one Allen requested.
- Sue always thought that if she were to murder someone it would be with a Revolver, so she was pleased that she had the card.
- Erin asked Bob is he had the Library, Rope, and Prof Plum. Bob was only able to show the suspect.
- Allen is starting to think that it was Mr. Green with the Wrench in the Conservatory. However, Allen is very bad at Clue and none of his guesses were right (especially since he has Mr. Green's card!).
- Erin is starving and is reminded of this every time she looks at her Kitchen card, one of her three room cards.
- Sue has two suspect cards.
- Allen is very confused about how one would use the Lead Pipe to kill someone. He thinks that you would use it to give them lead poisoning. He has that card.
- Sue wonders why there is a room that is the Hall (she has the card). Wouldn't all the space between the rooms be the hall?
- Allen asked Bob if he had Mrs. White, Rope, and Ballroom. Bob lies and says that he has none of them even thought he does possess the Ballroom but none of the others.
- Bob is pleased that he has the Candlestick card because he has an affinity for shiny objects. He also has the Wrench, but his feelings for that card are not the same.
- Sue was reminded by her Dining Room card that she was 45 minutes late for the party she was hosting, and had to leave, thus ending the game without any of them knowing who did it....

Suspect $\qquad$

Weapon: $\qquad$ Room: $\qquad$
4.4) THE PRICE IS RIGHT There are 5 contestants on The Price is Right, Bob, Betty, Sue, Tom, and Lindsay. Each of the contestants played a different game from the following games: Now and Then (pricing grocery items), Cliff Hangers (you "climb" a mountain), Plinko (drop discs down a board to land in different slots at the bottom), Master Key (open a lock with a key), and Dice Game (roll large dice to win a car). The contestants' ages are 20, 30, 40, 50, or $\mathbf{6 0}$ and they won nothing, a bedroom set, a car, a trip to the Bahamas, or $\mathbf{\$ 1 0 , 0 0 0}$. Use the following clues to fill in the grid.

- It's Bob's $60^{\text {th }}$ birthday today.
- Betty thought that shaving cream cost $\$ 3.29$ now, but the real price was $\$ 2.67$ so she lost her game.
- Sue loves rock climbing so her game fit her perfectly.
- Lindsay won 5 discs for her game by really only needed 1 since she landed on the $\$ 10,000$ on her first drop.
- Betty is the youngest contestant.
- Tom rolled a 2 and thought that his prize was higher than what he rolled. He was right and won!
- Lindsay is halfway to Bob's age.
- Sue did not win the bedroom set.
- Tom is the second oldest contestant.

| Name | Game | Winnings | Age |
| :---: | :---: | :---: | :---: |
| Bob |  |  |  |
| Betty |  |  |  |
| Sue |  |  |  |
| Tom |  |  |  |
| Lindsay |  |  |  |

4.5) France, China, Austria, and South Korea were awarded gold (either 6, 7, 8, or 5), silver (either $\mathbf{9}, \mathbf{3}, \mathbf{7}$, or $\mathbf{4}$ ), and bronze (either $5,4,9$, or 3 ) medals. Figure out how many of each type of medals were won by each of the four countries. Use the clues to figure out the number of medals awarded to each country.


Austria won the most gold medals.
South Korea won the fewest bronze medals.
Austria won more than seven silver medals.
South Korea won either 3 or 9 silver medals.
China won either five or six gold medals.
South Korea won a total of thirteen medals.
China won a total of nineteen medals.
One country won an odd number of bronze and nine silver.

|  | gold medals | silver medals | bronze medals |
| :---: | :--- | :--- | :--- |
| France |  |  |  |
| China |  |  |  |
| Austria |  |  |  |
| South <br> Korea | 13 |  |  |

4.6) Minute To Win It is a game show on which contestants have to try to do challenge in less than one minute. The challenges use common household objects and you work up to possibly winning $\$ 1$ million. Tommy was a contestant on Minute To Win It and he ended up being extremely good at the game and won $\$ 1$ million! Using the clues below, find out the following elements:

- what challenges he did: Balance the Bulb, Bobblehead, Movin' On Up, Back Flip, Uphill Battle, Keep It Up, Sticky Situation, Egg Tower, Bottoms Up
- what level the challenges were (their order): 1, 2, 3, 4, 5, 6, 7, 8, 9
- his winning time for each challenge: 27, 32, 39, 41, 43, 46, 52, 58, 60
- what household item he used for the challenge: Pedometer, Yo-Yo, Cups, Bread, Marbles, Pencils, Feathers, Paper Towel, Salt

1. The $1^{\text {st }}$ challenge wasn't his fastest time.
2. The challenge that used Pencils was definitely not the fastest time either but was faster than the Bottoms Up challenge.
3. The challenge that took the longest required using marbles.
4. Movin' On Up, Bobblehead, and the $4^{\text {th }}$ challenge all took less than 40 seconds.
5. Tommy was familiar with Pedometers which made the second challenge the fastest.
6. Tommy got his $4^{\text {th }}$ fastest time doing the Keep It Up challenge.
7. Back Flip was the second to last challenge right after Bottoms Up and right before the challenge that used Marbles.
8. Uphill Battle did not use Paper Towels.
9. It was really hard for Tommy to Balance the Bulb on Salt in the $4^{\text {th }}$ challenge.
10. The Cups challenge which was his $1^{\text {st }}$ challenge was done in 32 seconds.
11. Pencils were not part of the Keep It Up challenge but were used in the challenge that took 52 seconds.
12. Sticky Situation was difficult and took more time than Keep It Up but less than Back Flip.
13. Movin' On Up took more than 30 seconds.
14. The $6^{\text {th }}$ challenge used Feathers and took 41 seconds.
15. Sticky Situation took over 45 seconds but was faster than the challenge that took 52 seconds.
16. The $3^{\text {rd }}$ challenge used Bread and took 46 seconds.
17. Bottoms Up took the second longest times and did not use Paper Towels.

| level | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| challenge |  |  |  |  |  |  |  |  |  |
| time |  |  |  |  |  |  |  |  |  |
| object |  |  |  |  |  |  |  |  |  |

## More <br> Logic Puzzles

In this project, your group is going to try writing your own logic puzzle. It can be short and sweet; just make sure you have at least 3 choices for each of 3 categories. To write your own logic puzzle, it seems best to start with your solution and go backwards to give clues. When you're done, you will be switching with another group so make sure you have a legible copy at the end.

|  |  |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

5.1) MONOPOLY Aaden, Rachel, Karen, Colin, Laura, Scott, Joel, and Billy are playing a leisurely afternoon game of Monopoly. They each have a different piece to play with, a wheelbarrow, a thimble, a ship, a cannon, a moneybag, a racecar, a dog, and an iron. Figure out what piece each player has and where that piece is located on the board.

| Free Parking | RED Kentucky (\$220) | Chance | RED <br> Indiana <br> (\$220) | RED <br> Illinois <br> (\$240) | B \& O Railroad (\$200) | YELLOW Atlantic (\$260) | YELLOW Ventnor (\$260) | Water Works (\$150) | YELLOW Marvin Gardens (\$280) | Go To Jail |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ORANGE New York (\$200) | One piece is on a railroad.  <br> The cannon is directly next to the Luxury Tax space. GREEN <br> Pacific (\$300)  |  |  |  |  |  |  |  |  |  |
| ORANGE <br> Tennessee (\$180) | Park Place and Mediterranean Ave are not occupied. Colin has the piece that starts with the same letter as his first name. Billy's piece is related to his love for cash. |  |  |  |  |  |  |  |  | GREEN <br> North Carolina (\$300) |
| Community Chest | Karen bought her property for $\$ 320$. <br> Laura just missed her favorite utility but has yet to avoid a confrontation with a police |  |  |  |  |  |  |  |  | Community Chest |
| ORANGE <br> St. James (\$180) | officer. ${ }_{\text {Scott's piece must have caused him to speed and was punished. }}$ |  |  |  |  |  |  |  |  | GREEN Pennsylvania (\$320) |
| Pennsylvania RR (\$200) | Billy's piece is between Rachel and Laura's piece. <br> Karen loves the color green and her purchase reflects this. |  |  |  |  |  |  |  |  | $\begin{gathered} \text { Shortline RR } \\ (\$ 200) \end{gathered}$ |
| MAGENTA Virginia (\$160) | Despite his dislike of gardening, Joel picked his piece last and ends up with a gardening tool. |  |  |  |  |  |  |  |  | Chance |
| MAGENTA <br> State (\$140) | The railroad where the piece is located is in the same row as Water Works The person with the thimble is directly next to the free parking space. The thimble and the dog are on the same row as the moneybag. |  |  |  |  |  |  |  |  | DARK BLUE Park Place (\$350) |
| Electric Company (\$150) | Joel's property is more than \$130 and less than \$150. |  |  |  |  |  |  |  |  | Luxury Tax |
| MAGENTA St. <br> Charles (\$140) | A girl has the ship piece. <br> Aaden only has $\$ 60$ to buy a property. |  |  |  |  |  |  |  |  | DARK BLUE Boardwalk (\$400) |
| JAIL | LT BLUE Conneticut (\$120) | LT BLUE Vermont $(\$ 100)$ | Chance | LT BLUE Oriental (\$100) | Reading Railroad (\$200) | Income Tax | PURPLE Baltic (\$60) | Community Chest | PURPLE Mediterranean (\$60) | $\begin{aligned} & \text { GO (collect } \\ & \$ 200 \text { ) } \end{aligned}$ |

PIECE LOCATION

Aaden

Rachel $\qquad$
$\qquad$

Karen $\qquad$
$\qquad$

Colin $\qquad$
$\qquad$

Laura $\qquad$
$\qquad$

Scott $\qquad$
$\qquad$

Joel

Billy $\qquad$

5.2) Eight heavyweight-boxing champions (Sultan, Mike, George, Muhammad, Ingemar, Max, Jess, and James) of different years $(1892,1915,1930,1959,1978,1994,1996$, and 2007) have recently decided to hold a tournament to see who is the Ultimate Heavyweight Champion of the World. Before they held the competition, they wanted to see what their chances of winning were based on how many matches they've competed in $(\mathbf{2 4}, \mathbf{2 5}, \mathbf{2 8}, \mathbf{3 5}, \mathbf{5 8}, \mathbf{6 1}, \mathbf{7 0}$, and $\mathbf{8 1})$ and how many matches they've won $(\mathbf{1 6}, \mathbf{2 2}, \mathbf{2 6}, \mathbf{2 6}, \mathbf{5 0}, \mathbf{5 6}, \mathbf{5 6}$, and $\mathbf{7 6}$ ). From the clues below, figure out each competitor's last name (Johansson, Foreman, Ibragimov, Willard, Corbett, Ali, Schmeling, and Tyson), their first name, the number of matches in total they've competed in, and the number of matches they've won.

Muhammad and Max have both won the same number of matches. So have Johansson and Willard.
In 1892, the champion won 16 matches. In 1930, the champion competed in 70 matches.
Muhammad Ali was named the heavy-weight champ in 1978 and won 56 matches.
Tyson won in the early 90 's and had a penchant for ears.
Mike won 50 matches 35 years after Foreman who won 76.
Sultan's last name and Johansson's first name begin with an 'I'.
James was the first to win out of the group and Ingemar was the most recent.
Max's last name is not Willard, but a man whose first name starts with a " J " does have that last name.
The champ with the last name identical to a meat company has the first name of Mike and he has competed in 33 more matches than the first competitor.
The man with the last name of Foreman and the most number of matches competed in is the same man who sells miniature grills.
The man with the initials J.W. comes directly in between James and Max, and has won 26 matches. He has also competed in 10 more matches than Corbett.
James Corbett competed in 25 matches.
Sultan has competed in 1 less match than Corbett.
Ingemar has competed in an even number of matches.

| year |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| first name |  |  |  |  |  |  |  |  |
| last name |  |  |  |  |  |  |  |  |
| \# matches |  |  |  |  |  |  |  |  |
| \# wins |  |  |  |  |  |  |  |  |

Group \#6

| 1 | 2 | 4 | 3 |
| :--- | :--- | :--- | :--- |
| 3 | 4 | 1 | 2 |
| 4 | 3 | 2 | 1 |
| 2 | 1 | 3 | 4 |

Sudoku Puzzles

A Sudoku puzzle is a game of logic so reasoning skills and concentration are useful to complete one. The rules for a Sudoku are as follows: the digits 1 through the number of rows/columns can only appear once in every row, column and shaded or unshaded box. The clues are the digits already placed in the puzzle. Also remember: DON'T GUESS!!!

|  | 5 | 9 |  |  |  | 3 | 8 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 |  |  | 9 |  | 1 |  |  | 6 |
| 8 |  |  |  | 7 |  |  |  | 2 |
| 9 |  |  |  |  |  |  |  | 3 |
| 1 |  |  |  |  |  |  |  | 5 |
|  | 2 |  |  |  |  |  | 7 |  |
|  |  | 1 |  |  |  | 8 |  |  |
|  |  |  | 5 |  | 6 |  |  |  |
|  |  |  |  | 2 |  |  |  |  |

6.1) In a sudoku puzzle, some of the numbers are given and the goal is to fill in the blank cells. Consider the puzzle below.

6.2) Fill in the following sudoku puzzles.
a)

| 1 | 2 |  |  |
| :--- | :--- | :--- | :--- |
| 3 | 4 |  |  |
|  |  | 1 | 2 |
|  |  | 3 | 4 |

c)

b)

|  | 2 |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  | 3 |
| 1 |  |  |  |
|  |  | 4 |  |

d)

|  |  |  | 3 |
| :--- | :--- | :--- | :--- |
|  | 1 |  |  |
| 1 |  |  |  |
|  |  | 2 |  |

Here are some hints:
a) There must be a 1 somewhere in the third row, and the only empty cells in the row are in the 2nd and 4th columns. Why can't the 1 go in the 4th column? Fill in the entire third row.
b) Next consider the lower right-hand box, which now needs a 1 and a 2. There is only one choice for the 1 (why?). Fill in the remainder of the grid.
6.3) Now try a 6 by 6 sudoku puzzle.
a) One with numbers

b) The other with BUTLER


Do you notice a similarity with these puzzles?
6.4) The most common size of sudoku is a $9 \times 9$.

| 2 |  |  |  | 1 |  | 6 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 4 |  | 3 | 8 |  |  | 5 | 2 |
| 1 |  | 5 |  | 4 | 6 | 7 | 9 |  |
| 9 |  | 8 |  | 5 | 2 | 3 | 4 |  |
|  | 3 |  | 4 | 6 |  |  | 1 | 8 |
|  | 1 |  |  |  | 3 |  | 7 |  |
| 8 | 2 | 4 |  |  | 1 | 5 |  |  |
|  |  |  | 6 | 7 |  |  |  |  |
| 7 | 6 | 1 |  |  | 8 | 4 |  |  |

One strategy to try is to identify an empty cell and determine which numbers could fill it. For example, the cell in the first column of the second row is empty. The row contains the numbers $2,3,4,5$, and 8 ; the column contains $1,2,7,8$ and 9 ; and the box contains 1 , 2,4 , and 5 . The only number not in any of the three lists is 6 , so that number must go into that cell.

Another strategy is to choose a number missing from a row, column, or box and try to place it correctly. For example, the third row needs a 3 and has blank cells in the 2 nd, 4 th, and 9 th rows. There is already a 3 in columns 2 and 4 , so the 9 th cell in the third column must be a 3 . Continue with these and other strategies you find to fill in the remainder of the puzzle
6.5) Fill in the $6 \times 6$ Sudoku.

|  |  | 6 | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4 |  | 5 |  | 1 | 6 |
|  |  | 1 | 6 |  |  |
|  |  | 2 | 5 |  |  |
| 5 | 1 |  |  |  | 2 |
|  |  | 4 | 1 |  |  |

6.6) Fill in the $9 \times 9$ Sudokus.

| 5 |  | 3 | 8 | 1 | 9 |  | 2 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 1 |  | 4 |  |  | 3 | 9 | 5 |
|  | 4 | 2 |  |  |  | 7 | 8 | 1 |
| 6 |  | 8 | 3 |  | 1 |  | 7 |  |
|  |  |  |  |  |  |  |  | 8 |
| 1 |  | 9 | 5 | 7 |  |  | 4 | 6 |
| 2 |  |  |  | 8 |  | 9 | 5 | 3 |
|  |  | 5 |  | 9 | 6 |  |  |  |
| 4 |  |  | 7 |  | 5 | 8 | 6 | 2 |


| 2 |  |  |  | 1 |  | 6 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 4 |  | 3 | 8 |  |  | 5 | 2 |
| 1 |  | 5 |  | 4 | 6 | 7 | 9 |  |
| 9 |  | 8 |  | 5 | 2 | 3 | 4 |  |
|  | 3 |  | 4 | 6 |  |  | 1 | 8 |
|  | 1 |  |  |  | 3 |  | 7 |  |
| 8 | 2 | 4 |  |  | 1 | 5 |  |  |
|  |  |  | 6 | 7 |  |  |  |  |
| 7 | 6 | 1 |  |  | 8 | 4 |  |  |

## SETS and VENN DIAGRAMS NOTES

SET: any group or collection of well-defined objects called elements, denoted by listing the elements inside set braces \{ \}
Set or not?
i) The NFL teams that start with the letter V. Yes No
ii) a, b, c Yes No
iii) The set of all good ping pong players. Yes No
iv) $\{1,2,4,6,8\} \quad$ Yes No

Definitions: $\varnothing=\{ \}$ Empty or Null Set $=$ the set containing no elements
$\in$ "is an element of"
ex. $1 \in\{1,2,3\}$
$\notin$ "is not an element of" ex. $4 \notin\{1,2,3\}$
= "is equal to"; $\mathrm{A}=\mathrm{B}$ if A and B have exactly the same elements
$\subseteq$ "is a subset of"; $A \subseteq B$ if every element in A is in B , subsets always use set braces ex. $\{1,2\} \subseteq\{1,2,3\}$
$\nsubseteq$ "is not a subset of" ex. $\{1,4\} \nsubseteq\{1,2,3\}$

Every set A has $\qquad$ and $\qquad$ as a subset.

Ex1. Fill in the blanks with $\in, \notin,=, \subseteq$, or $\not \subset$.
a) $\qquad$ $\{a, b, c\}$
b) $a$ $\qquad$ $\{a, b, c\}$
c) $\{a\} \_$_ $\{a, b, c\}$
d) $d \ldots \quad\{a, b, c\}$
e) $\{d\} \ldots \quad\{a, b, c\}$
f) $\{b, c, a\} \quad\{a, b, c\}$
$\cap$ AND intersection $A \cap B$ is the set of all elements in both $A$ and $B$, i.e. what's in common ex. $\{1,2,3,4\} \cap\{1,3,5\}=$
$\cup$ OR union $A \cup B$ is the set of all elements in either A or B or both, i.e. both sets together ex. $\{1,2,3,4\} \cup\{1,3,5\}=$
universal set $U$ is the set of elements being considered (the big set)
complement $A^{c}$ of a set A is the set of elements of the universal set U that are not in A .
ex. If $U=\{1,2,3,4,5,6,7\}$ and $A=\{3,4,5,6\}$, then $A^{c}=$ $\qquad$ .

Note: $U^{c}=$ $\qquad$ and $\varnothing^{c}=$ $\qquad$

Ex2. Given $A=\{2,4,6,8\}, B=\{1,3,5,7\}, C=\{1,2,3,4,9\}$, and $\mathrm{U}=\{1,2,3,4,5,6,7,8,9,10\}$, determine
a) $A \cap B=$ $\qquad$
b) $A \cup B=$ $\qquad$
c) $C^{c}=$ $\qquad$ d) $B \cap C=$ $\qquad$
e) $A \cup(B \cap C)=$ $\qquad$
f) $(A \cup B) \cap C^{c}=$ $\qquad$
cardinality of set $\mathbf{A} n(A)=$ the number of elements in set A
ex. If $A=\{a, b, c\}, B=\varnothing$, and $C=\{0\}$, then $n(A)=$ $\qquad$ , $n(B)=$ $\qquad$ , $n(C)=$ $\qquad$ .

VENN DIAGRAMS are a pictorial way of demonstrating sets.
The box as a whole represents all members of the universal set. Intersections are shown by overlapping circles which represents subsets.


Ex3. A survey of college students asked if they play intramural basketball (B), softball (S), or soccer (C). Shade the areas representing and write in set notation:
a) basketball \& soccer

b) softball or soccer

c) basketball \& softball, but not soccer


INCLUSION-EXCLUSION PRINCIPLE $n(A \cup B)=n(A)+n(B)-n(A \cap B)$


Ex4. If $n(L)=780, n(M)=240, n(L \cup M)=970$, find $n(L \cap M)=$ $\qquad$

Ex5. If 20 students like chemistry, 30 like math, and 10 like both chemistry and math, how many like chemistry or math?

Ex6. A survey of $8004^{\text {th }}$ graders indicates that 250 said their family owned a Wii game system, 420 own Nintendo DS, and 180 own both a Wii and a DS. Draw a Venn Diagram and answer the following: how many $4^{\text {th }}$ graders in the survey:
a) own a Wii or a DS?
b) own neither a Wii nor a DS?

c) own a DS, but not a Wii?

Ex7. 1000 households were asked how they got their sports scores:
280 internet
740 ESPN
545 newspaper
410 ESPN and newspaper 185 ESPN and internet 105 ESPN, paper, and internet 65 internet, but not ESPN or paper How many got their scores from:
a) ESPN, but not paper or internet?
b) paper but not ESPN or internet?
c) ESPN or paper?
d) none of the three?

## THE EMPTY SET

There is an interesting quotation from Don Marquis' Archy and Mehitabel (The Merry Flea). It seems quite appropriate when first introducing the idea of the empty set. Mr. Marquis writes: "What is there in a vacuum to make one afraid?" said the flea. "There is nothing in it," I said, "and that is what makes one afraid to contemplate it. A person can't think of a place with nothing at all in it without going nutty, and if he tries to think that nothing is something after all, he gets nuttier." Archy's analysis of this situation is similar to that of some students. The following questions may add to your bewilderment but is worth pondering!

Circle True or False for each statement.
a) $\varnothing \in \varnothing \quad \mathrm{T} \quad \mathrm{F}$
b) $\varnothing \subseteq\{0\}$
T
F
c) $\varnothing=\{0\} \quad \mathrm{T} \quad \mathrm{F}$
d) $\varnothing \subseteq\{\varnothing\} \quad \mathrm{T} \quad \mathrm{F}$
e) $\varnothing=\varnothing \quad$ T $\quad$ F
f) $\varnothing=\{\varnothing\} \quad \mathrm{T} \quad \mathrm{F}$
g) $\varnothing \subseteq \mathrm{A}$ for all sets $\mathrm{A} \quad \mathrm{T} \quad \mathrm{F}$
h) $\varnothing \subseteq \varnothing \quad$ T F
i) $\varnothing \in\{\varnothing\} \quad \mathrm{T} \quad \mathrm{F}$
j) $\varnothing=0 \quad \mathrm{~T} \quad \mathrm{~F}$

## Set Homework Problems:

1) Given $U=(1,2,3,4,5,6,7,8,9\}, A=\{2,3,4,5\}, B=\{1,7\}$, and $C=\{9\}$, determine the following:
a) $A \cup B$
b) $A \cap B$
c) $A^{C}$
d) $n(A \cup C)$
e) $(A \cup B)^{C} \cup C$
f) $\emptyset \nsubseteq U$ true false
g) $7 \in B$ true false
h) $\{7\} \in B$ true false
2) Given the universal set $U=\{$ Al, Bob, Dan, Ed, Jan, Kim, Sue $\}$ and the set of those who know how to swim $S=\{B o b, E d$, Jan, $\operatorname{Kim}\}$ and the set of those who know how to skateboard $B=\{$ Al, Kim, Sue $\}$, find the elements of the indicated sets.
a) $S \cap B$
b) those who can swim or skateboard
c) $B^{C}$
d) those who cannot swim
e) $S \cup B$
f) those who can skateboard and swim
g) $S^{C} \cap B$
h) those who can swim or cannot skateboard
i) those who can neither swim nor skateboard
3) Given the universal set $\mathrm{U}=$ \{red, blue, green, orange, yellow, purple, white $\}$ and the following race car paints: $\operatorname{car} \mathrm{A}=\{$ red, blue $\}$, car $\mathrm{B}=\{$ green, orange, red $\}$, and car $\mathrm{C}=\{$ orange, white, red $\}$, determine the indicated sets.
a) $A \cap B \cap C$
b) $(A \cap B \cap C)^{C}$
c) $A \cup(B \cap C)$
d) $A \cup B \cup C$
e) $(A \cap B)^{C} \cap C$
f) $(A \cup B \cup C)^{C}$
4) Given the Venn diagram and $n(U)=25$, determine the following:
a) $n(A \cap B)$
b) $n(A \cup B)$
c) $n(A)$
d) $n(B)$
e) $n\left(A \cap B^{C}\right)$
f) $n\left(A^{C} \cup B\right)$
g) $n(A \cap B)^{C}$
h) $n(A \cup B)^{C}$

5) A survey of 1500 people in NYC showed that 1140 like the Yankees, 680 like the Mets, and 120 cheer for neither team. Draw a Venn diagram.
a) How many cheer for both teams?
b) How many cheer only for the Yankees?
6) A group of 32 play bridge every other Tuesday at the senior center (eight tables of four). If 17 of the bridge players like salty snacks, 25 like sweet snacks, and all of the players like at least one of the kinds of snacks; how many like both sweet and salty snacks? Draw a Venn diagram.
7) In a survey of 100 people, 41 reported that they had played Chutes \& Ladders when they were young and 65 reported that they played Candyland when they were young, and 25 had played both as youngsters. Draw a Venn diagram to determine the how many people:
a) played only Candyland?
b) played only Chutes \& Ladders?
c) played neither of these games?
d) did not play Candyland?
e) played Chutes \& Ladders or Candyland?
8) A certain number of students who read the Star newspaper were surveyed. The findings were: 37 will work the crossword puzzle, 32 will work the sudoku, 9 will work both the crossword puzzle and sudoku, and 12 will work neither puzzle. Draw a Venn diagram to determine how many will work:
a) crossword puzzle or sudoku?
b) only the sudoku?
c) not work the sudoku?
d) How many were surveyed?
9) Of Carmel High School's varsity athletes, 30 play soccer, 25 play hockey, 40 play football, 6 play soccer and hockey, 8 play hockey and football, 9 play football and soccer, and 4 play all three sports. Draw a Venn diagram to determine the how many athletes:
a) play only soccer?
b) play soccer or hockey?
c) play football or hockey?
d) play only one of these sports?
e) play football, hockey, or soccer?
10) An activity director for a cruise ship has surveyed 240 passengers. Of the 240 passengers, 135 like swimming, 150 like dancing, 65 like games, 80 like swimming and dancing, 40 like swimming and games, 25 like dancing and games, and 15 like all three activities. Draw a Venn diagram. How many passengers:
a) like exactly 2 of 3 activities?
b) like only swimming?
c) like none of these activities?
11) A survey of 770 dentists office indicates that 305 subscribe to Golf magazine, 290 to Field and Stream (FS), 390 to Sports Illustrated (SI), 110 to Golf and FS, 135 to FS and SI, 150 to Golf and SI, and 85 to all three magazines. Draw a Venn diagram. How many of the surveyed dentist offices subscribe to:
a) exactly one of these magazines?
b) only SI?
c) exactly two of these magazines?
d) none of these?

## Group \#7


7.1) Take your groups deck of card and organize it in suits from ace, $2,3, \ldots 10, \mathrm{~J}, \mathrm{Q}, \mathrm{K}$ and determine the following:
a) Number of cards in a deck:
b) List the suits (name and symbol) along with their color:
c) List the cards in each suit (also known as ranks):
d) List the ranks considered face cards:
e) Number of cards of each rank:
f) Number of face cards:
g) Number of red (or black) cards:
h) Number of clubs (or any other single suit):
7.2) Now find the cards in your groups deck, make an abbreviated list of them, and determine how many cards are contained in the set containing:
a) $\mathrm{A}=\{$ spades or aces $\}=\cup$ ace
b) $\mathrm{B}=\{$ threes or sixes $\}=3 \mathrm{U} 6$
$\mathrm{n}(\mathrm{A})=$ $\qquad$
$\mathrm{n}(\mathrm{B})=$ $\qquad$
c) $\mathrm{C}=\{$ face or black $\}=$ face $U$ black

$$
\mathrm{n}(\mathrm{C})=
$$

d) $\mathrm{D}=\{$ clubs and twos $\}=\cap 2$

$$
\mathrm{n}(\mathrm{D})=
$$

$\qquad$
e) $\mathrm{E}=\{$ aces or eights $\}=\operatorname{aces} \cup 8$

$$
\mathrm{n}(\mathrm{E})=
$$

f) $\mathrm{F}=\{$ face and diamonds $\}=$ face $\cap \star$

$$
\mathrm{n}(\mathrm{~F})=
$$

$\qquad$
g) $\mathrm{G}=\{$ jacks and face cards $\}=\mathrm{J} \cap$ face
h) $\mathrm{H}=\{$ face and black $\}=$ face $\cap$ black

$$
\mathrm{n}(\mathrm{G})=
$$

$$
\mathrm{n}(\mathrm{H})=
$$

$\qquad$
i) $\mathrm{I}=\{$ queens or face cards $\}=\mathrm{Q} \cup$ face $\quad$ j) $\mathrm{J}=\{$ aces and eights $\}=$ ace $\cap 8$
k) $\mathrm{K}=\{$ not hearts nor spades $\}=(\vee \cup \boldsymbol{Q})^{\mathrm{C}}$

$$
\mathrm{n}(\mathrm{I})=
$$

$\qquad$
$n(K)=$
$\mathrm{n}(\mathrm{L})=$ $\qquad$
m) $\mathrm{M}=\{$ sevens or kings, but not clubs $\}=(7 \cup \mathrm{~K}) \cap \mathfrak{c}^{\mathrm{C}}$
n) $\mathrm{N}=\{$ nines or red, but not kings $\}=(9 \mathrm{U}$ red $) \cap \mathrm{K}^{\mathrm{C}}$

$$
\mathrm{n}(\mathrm{M})=
$$

$\mathrm{n}(\mathrm{N})=$ $\qquad$
7.3) Now we will investigate how many subsets are possible in a set with a certain number of elements. For each of the given sets, list all possible subsets, starting with the zero element subset, then one element subset(s), and so on.
a) $\mathrm{A}=\{$ three of spades $\}=\{3 \boldsymbol{\Delta}\}$
b) $\mathrm{B}=\{$ jack of diamonds, two of hearts $\}=\{\mathrm{J}, 2 \vee\}$
c) $\mathrm{C}=\{$ four of clubs, queen of hearts, ace of spades $\}=\{4 \boldsymbol{\&}, \mathrm{Q} \boldsymbol{\bullet}, \mathrm{A} \boldsymbol{\wedge}\}$
d) $\mathrm{D}=\{$ five of diamonds, king of clubs, seven of diamonds, 10 of hearts $\}=\{5 \star, \mathrm{~K} \boldsymbol{\imath}, 7 \downarrow, 10 \bullet\}$
e) Now count your subsets:
$\qquad$
of $\mathrm{A}=$
of $B=$ $\qquad$
$\qquad$ of $D=$ $\qquad$
f) Make a speculation as to how many subsets a set with $\mathbf{n}$ elements would have.
7.4) You have some friends over for a friendly poker tournament.
a) Being the host with the most you offer them each a beverage. They all get water, but they can decide if they want ice, lemon slice, and/or a straw. How many different drinks are you actually offering them?
b) You all decide to order in pizza. The pizza place offers 8 different toppings and advertises that they offer over 250 different pizzas (assuming no double toppings). Are they correct?
c) What is the minimum number of toppings that this pizza place must have available if it wants to offer at least 1000 different types of pizza?
d) Another pizza places advertises 16384 different pizzas. How many different toppings do they offer?
7.5) Hoyle (a company that produces playing cards) did a survey of card players. They decided to use the following Venn diagram to help illustrate the results of their survey. $\mathrm{S}=$ \{those who like to play solitaire $\}, \mathrm{P}=\{$ those who like to play poker $\}$, $B=\{$ those who like to play bridge $\}$. Shade in the regions that represent and describe using set notation:
a) those who like to play poker, but not solitaire

b) those who like bridge and solitaire
c) those who don't like poker and do not like bridge (same as those who don't like poker or bridge)


Set notation: $\qquad$
$\qquad$
$\qquad$
d) Using the same sets, S, P, and B, describe the shaded region and then put it in set notation (using unions, intersections, complements, etc).

i) describe in words:

ii) describe in words:

iii) describe in words:
$\qquad$
$\qquad$ set notation $\qquad$
7.6) Determine the set notation and the number of cards in each set.
a) $\mathrm{A}=\{$ fives or face $\}=$ $\qquad$ b) $\mathrm{B}=\{$ hearts and face $\}=$ $\qquad$

$$
\mathrm{n}(\mathrm{~A})=
$$

$$
\mathrm{n}(\mathrm{~B})=
$$

c) $\mathrm{C}=\{$ neither red nor clubs $\}=$ $\qquad$ d) $\mathrm{D}=\{$ red but not jacks $\}=$ $\qquad$

$$
\mathrm{n}(\mathrm{C})=
$$

$$
\mathrm{n}(\mathrm{D})=
$$

e) $\mathrm{E}=\{$ face and black, but not queens $\}$

$$
\begin{aligned}
& = \\
& n(E)=
\end{aligned}
$$

f) $\mathrm{F}=\{$ threes or spades but not hearts $\}$
$\qquad$
$\mathrm{n}(\mathrm{F})=$ $\qquad$
7.7) $\mathrm{S}=\{$ those who like to play solitaire $\}, \mathrm{P}=\{$ those who like to play poker $\}, \mathrm{B}=\{$ those who like to play bridge $\}$. Shade in the Venn diagram and describe the set using set notation.
a) those who like poker or solitaire

b) those who don't like poker

c) those who like bridge but not poker


Set notation: $\qquad$
$\qquad$
$\qquad$
d) Using the same sets, S, P, and B, describe the shaded region and then put it in set notation (using unions, intersections, complements, etc).

i) describe in words:

ii) describe in words:
set notation $\qquad$
Group \#8

set notation $\qquad$ set notation $\qquad$

## Venn Diagrams

Draw Venn diagrams to answer the following:
8.1) Find $n(S \cap T)$, given that $n(S)=4, n(T)=12$, and $n(S \cup T)=15$.
8.2) Find $n(T)$, given that $n(S)=14, n(S \cap T)=6$, and $n(S \cup T)=17$.
8.3) In a class of 50 students, 18 play badminton, 26 play lawn bowling, and 2 play both badminton and lawn bowling. How many students in the class do not play either badminton or lawn bowling?
8.4) In a school of 320 students, 85 students are in the intramural kickball, 200 students are on intramural dodgeball, and 60 students participate in both activities. How many students are involved in either kickball or dodgeball?
8.5) Given $\mathrm{F}=$ a set of 47 kids who like Freeze Tag and $\mathrm{T}=\mathrm{a}$ set of 25 kids who like TV Tag, determine each of the following by drawing a Venn diagram for each of the situations:
a) The maximum possible number of these kids who like Freeze Tag or TV Tag.
[ i.e. largest value for $n(F \cup T)$ ]

b)


The minimum possible number of these kids who like Freeze Tag or TV Tag.
[ i.e. smallest value for $n(F \cup T)$ ]
c) The maximum possible number of these kids who like both Freeze and TV Tag.
[i.e. largest value for $n(F \cap T)$ ]

d)


The minimum possible number of these kids who like both Freeze and TV Tag.
[i.e. smallest value for $n(F \cap T)$ ]
go to a barbeque with 26 guests and discover that 14 enjoy Frisbee, 10 enjoy Jumping
8.6) You Rope, and 5 enjoy Hopscotch. Four enjoy Frisbee and Jumping Rope, 3 enjoy Frisbee and Hopscotch, and one enjoys Jumping Rope and Hopscotch. If no one enjoys all three types of activities, how many guests do not enjoy any of these activities?
8.7) Twenty-four balls are in a bin in the backyard. Twelve of the balls are black, six of the balls are flat, and fifteen of the balls have team logos on them. There is only one ball that is black and flat and has a logo. Two of the balls are black and flat and do not have logos. Two of the balls are flat and have logos but are not black. If all of the balls in the bin have at least one of the mentioned characteristics, how many balls are black and have logos but are not flat?
8.8) Illustrate the following on a Venn diagram:

$$
\begin{gathered}
n(A \cap B)=6 \quad n(A \cap B \cap C)=4 \quad n(A \cap C)=7 \\
n(B \cap C)=4 \quad n\left(A \cap C^{C}\right)=11 \\
n\left(A^{C} \cap B^{C} \cap C^{C}\right)=5 \quad n(C)=15 \\
n(B)=12
\end{gathered}
$$


8.9) Fill in the Venn diagram with the following information and then answer the questions about children attending a HUGE birthday party:

120 are girls
150 like Pin the Tail on the Donkey
170 like Musical Chairs
100 of the boys don't like Musical Chairs
108 of the boys like Pin the Tail on the Donkey
18 of the girls who don't liked Pin the Tail on the Donkey, like Musical Chairs
78 boys who don't like Pin the Tail on the Donkey don't like Musical Chairs either
30 of the girls who liked Pin the Tail on the Donkey also like Musical Chairs

a) How many kids are at this birthday party?
b) How many attendees do not like Pin the Tail?
c) How many girls do not like Musical Chairs?
d) How many boys who like Musical Chairs like Pin the Tail on the Donkey?
8.10) Given that $\mathrm{C}=$ the set of kids who want to play Capture the Flag
$\mathrm{R}=$ the set of kids who want to play Red Light Green Light $\mathrm{F}=$ the set of kids who want to play Four Square
Draw a Venn diagram for each to determine the following:
a) The maximum number of kids who want to play Capture the Flag or Red Light Green Light or Four Square. [i.e. largest value for $n(C \cup R U F)$ ]

b) The minimum possible number of kids who want to play
 Capture the Flag or Red Light Green Light or Four Square. [i.e. smallest value for $n(C \cup R U F)$ ]
c) The maximum possible number of kids who want to play Capture the Flag and Red Light Green Light and Four Square. [i.e. largest value for $n(C \cap R \cap F)$ ]

d) The minimum possible number of kids who want to play Capture the Flag and Red Light Green Light and Four Square.
[i.e. smallest value for $n(C \cap R \cap F)$ ]
8.11) In a recent survey, consumers were asked where they shopped for sports equipment. The following results were obtained: 213 shopped at Dick's Sporting Goods, 294 shopped at Play It Again Sports, and 337 shopped at Cabelas, 198 shopped at Play It Again and Cabelas, 382 shopped at Play It Again or Dick's, 61 shopped at Dick's and Cabelas but not at Play It Again, 109 shopped at all three, and 64 shopped at neither Dick's nor Play It Again nor Cabelas. How many consumers
a) shopped at more than one of the stores?
b) shopped exclusively at Cabelas?
c) shopped at Dick's and Play It Again, but not Cabelas?

8.12) A chicken farmer decided his chickens would participate in a game of Red Rover. In order to determine fair teams, the farmer surveyed his flock and found there were: (Assume that red and brown are opposites in the chicken world and note that hens are female chickens, roosters are male chickens.)

| 9 fat red roosters | 2 fat red hens |
| :--- | :--- |
| 26 fat roosters | 7 thin brown hens |
| 37 fat chickens | 6 thin red roosters |
| 18 thin brown roosters | 5 thin red hens |

How many chickens were:
a) red?
b) male?
c) fat, but not male?
d) brown, but not fat?

e) red and fat?
8.13) A survey of 136 arm chair athletes yielded the following information:

37 watched football
2 watch all 4
11 watch only football
6 wh
3 watch football, basketball, baseball, but not soccer
11 watch football, baseball, soccer

50 watched baseball
68 watched soccer
14 watch only basketball
21 watch both football and baseball
27 watch both baseball and soccer
1 watches football, basketball, soccer, but not baseball 12 watch basketball, baseball, soccer

How many of the surveyed armchair athletes watch no football, no basketball, no baseball, and no soccer (they watch other sports)?


## Group \#9



A group of people are gathered where $L$ is the set of all lacrosse players, $S$ is the set of all soccer players, and $B$ is the set of all baseball players.
9.1) State in words what each section of the Venn Diagram represents.
i)
ii)
iii)
iv)
v)
vi)
vii)
viii)
9.2) Using set notation describe the sections:
i)
ii)
iii)
iv)


GROUP WORK Your group will write 1 two circle and 2 three circle Venn diagram problems.
Be careful and make sure that you are giving enough information so your problem can be solved! (But don't make it too easy either.) Have one person write a clean copy so you can trade problems with another group.
9.3) Two circle Venn diagram problem.

9.4) Three circle Venn diagram problem.

9.5) Another three circle Venn diagram problem.

9.6) Let A and B by subsets of $\mathrm{U}=\{a, b, c, d, 1,2,3,4\}, \mathrm{A}=\{a, 2\}, \mathrm{A} \cap \mathrm{B}=\{2\}$, and $\mathrm{A} \cup \mathrm{B}=\{a, b, d, 2,4\}$. Find B .
9.7) Let $\mathrm{U}=\{a, b, c, d, e, f, g, h, i\}$ where $\mathrm{A}=\{a, c, f\}, \mathrm{A} \cap \mathrm{B}=\{f\}$, and $\mathrm{A} \cup \mathrm{B}=\{a, c, d, f, h\}$. Find B .
9.8) Let $U=\{1,2,3,4,5,6,7,8,9,10\}$. Let $A, B$, and $C$ be subsets where:
$A^{C}=\{2,4,6,8\}$
$\mathrm{B} \cup \mathrm{C}=\{2,3,5,6,7,8,9\}$
$\mathrm{B} \cap \mathrm{C}=\{7\}$
$\mathrm{A} \cap \mathrm{B}=\{3,5,7\}$
$A \cup B=\{1,2,3,5,7,9,10\}$
$B^{C} \cap C^{C}=\{1,4,10\}$
$\mathrm{A} \cap \mathrm{C}=\{7,9\}$.
a) Fill in the Venn diagram of $U$ and its subsets.

b) Find A.
c) Find B.
d) Find C.
9.9) Let $U=\{1,2,3,4,5,6,7,8,9\}$. Let $A, B$, and $C$ be subsets where
$A^{C}=\{1,2,6,7,9\}$
$A \cup B=\{2,3,4,5,6,8\}$
$\mathrm{B} \cup \mathrm{C}=\{2,3,5,6,7,8\}$
$\mathrm{B} \cap \mathrm{C}=\{6,8\}$
$B^{C} \cap C^{C}=\{1,4,9\}$
$A \cap C=\{3,8\}$.
a) Draw a Venn diagram of $U$ and its subsets.

b) Find A.
c) Find B.
d) Find C.

## Group \#10

## Is order important?



Discuss the following questions and decide if order makes a difference or not (just write YES if it does matter or NO). You don't have to figure out the actual answers (yet).
10.1) A university has three table tennis scholarships each worth $\$ 1000$. How many ways are there to award these after they've narrowed the candidates to 6 ping pong players?
10.2) This same university has three badminton scholarships, one worth $\$ 3000$, one worth $\$ 2000$, and one worth $\$ 1000$. If the field of qualified badminton candidates is also narrowed to 6 , how many ways are there to give the scholarships?
10.3) How many ways are there for 8 teams in a soccer league to end the season in $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ place?
10.4) How many ways are there for 7 teams in a football league to end the season in the top three?
10.5) After being dealt five cards, how many different ways can a player arrange the cards in their hand?
10.6) How many different foursomes can be formed from among 6 golfers?
10.7) How many different finishes can occur in an 8 -horse race (excluding the possibility of ties)?
10.8) How many different 9 player batting line ups are possible on a softball team with a 13 -player roster?
10.9) How many different ways are there for a soccer team of 11 to choose 3 players to be team captains?
10.10) How many different 5 card poker hands are there?
10.11) How many different solutions are there for the game of Clue if the game has 9 rooms, 6 weapons, and 6 suspects?
10.12) How many different outcomes are there when you flip a penny three times in a row?
10.13) How many different outcomes are then when you flip three identical nickels at the same time?
10.14) How many different blackjack hands are there (first card dealt down, second dealt face up)?
10.15) In how many ways can 7 go-karts be situated on a circular track during a race, assuming that no two are exactly side by side?
10.16) How many ways are there to seat 4 people around a table to play bridge?
10.17) A 3 reel slot machine has 11 different symbols on each reel including one JACKPOT symbol. How many different ways could you get no JACKPOTs?
10.18) How many ways are there to grab 3 marbles out of a bag with 6 red marbles, 5 green marbles, and 2 blue marbles?
10.19) How many ways are there to grab 3 marbles out of that same bag so that the first is red, the second is green, and the third is blue?
10.20) How many different 3 piece ski outfits could you come up with if you have 2 jackets, 5 hats, and 4 pairs of gloves?
10.21) There are 100 squares on a Battleship game board. How many ways could you start off the game during your first three shots?
10.22) Uno is played with a deck of 108 Uno cards and everyone is dealt 7 cards. How many different Uno hands are there?
10.23) A family of 4 is playing on the game show Family Feud. How many different ways could they sit in a row to be asked questions?
10.24) Sixteen marching bands have volunteered to perform at the Lucas Oil Stadium during half time of a Colts game, but there in only enough for 10 of the bands to be on the field. How many ways are possible for the bands to be chosen?
10.25) A coed softball team consists of 7 women and 6 men. How many different ways can 2 captains be picked if there must be one man and one woman?
10.26) A hockey league has 15 teams. If every team must play every other team once in the first round of league play, how many games must be scheduled?
10.27) How many different "words" (can be nonsense words) can be formed using all of the letters in BIKE?
10.28) Given a group of 20 cheerleaders, 15 females and 5 males, how many ways can a group of 10 be picked to perform in a competition if there must be 7 females and 3 males?
10.29) If you are rolling a single die, how many ways are there to roll exactly two 4 's in three rolls?
10.30) There are 10 marbles in a bag, 5 green, 3 blue, and 2 red and you want to grab 3 of them, one at a time without replacement. How many ways are there to reach in and grab a green, and then a blue and then another green?

## INTRO TO COMBINATORICS NOTES

COMBINATORICS is a branch of mathematics dealing with counting different outcomes of some task called an experiment.
A few definitions we will need in our study are as follows:
Experiment : an activity with an observable outcome Ex1. flip a coin Ex2. roll a die

Sample Space S : all of the outcomes of an experiment
Ex1. $\mathrm{S}=$
Ex2. $\mathrm{S}=$

Event E: subset of the sample space (some of the possible outcomes)
Ex1. event : roll a head $\mathrm{E}=\quad$ Ex2. event : roll an even $\mathrm{E}=$
$\mathbf{n}(\mathrm{A})=$ number of elements in the set A
Ex1. $\mathrm{n}(\mathrm{S})=$
$n(E)=$
Ex2. $\mathrm{n}(\mathrm{S})=$
$n(E)=$

Single-stage event: only one outcome (but may be several possible for this one outcome), for example Ex1, Ex2.

Multi-stage event: experiments having two or more stages at which different outcomes occur, for example Ex3, Ex4

## FUNDAMENTAL PRINCIPLE OF COUNTING

## I. Counting by Making a List

One way to count outcomes is to list the elements (outcomes), but multi-stage events can be cumbersome to list all possible outcomes and know you have them all.

Ex3. Roll a die and then toss a coin; event = getting an odd and a tail

$$
\mathrm{S}=
$$

Ex4. Flip a coin three times in a row; event $=$ tossing exactly two heads

$$
\begin{gathered}
\mathrm{S}= \\
\mathrm{E}=
\end{gathered}
$$

## II. Counting with a Tree Diagram

Yet another method to count outcomes that is a little more visual is to construct a tree diagram that can be especially useful for a multistage event such as Ex3 and Ex4. Make a tree diagram for the following experiments to find $\mathrm{n}(\mathrm{S})$.

Ex3.
Ex4.

## III. The Counting Principle

If you were asked to make a tree diagram for flipping a coin 10 times, it would get a little out of hand. The counting principle would work much better. The counting principle states that in a multi-stage experiment, you can multiply the number of possible choices (or outcomes) at each stage together to get the total number of possible outcomes.

Ex3. There are 6 outcomes for the first stage (rolling a die) and there are 2 outcomes for the second stage (flipping the coin), so there are (4)(2) $=8$ total outcomes.

Ex4. Flip a coin 3 times in a row. Total number of possible outcomes $=$
Let's look at coin flipping more:

$$
\begin{array}{cc}
S=4 \text { flips of a coin } & n(S)= \\
S=5 \text { flips of a coin } & n(S)= \\
\vdots & \\
S=n \text { flips of a coin } & n(S)=
\end{array}
$$

Ex5. You go out to dinner and have a choice of 6 appetizers, 10 main courses, and 5 desserts. How many different three course meals are possible?

Ex6. A lacrosse team is going cheap to number their jerseys- they found a bunch of iron-on 1 's, 2 's, 6 's, 7 's, and 8 's. How many two-digit numbers can be made if
a) repetition is allowed?
b) repetition is not allowed?
c) only odd two-digit numbers can be used (repetition not allowed)?

## HOMEWORK PROBLEMS:

1) How many three-digit numbers can be made from the digits $2,3,4,5,9$ if
a) repetition is allowed?
b) repetition is not allowed?
c) number must be even and repetition is not allowed?
2) A soccer store offers 4 choices of shin-guards, 8 different soccer shoes, and 5 different soccer balls for kids. How many different ways are there to outfit a child at this store?
3) A college is looking for a new football coach and a new swimming coach. In how many ways can the positions be filled if there are three applicants for the football position and four for the swimming position?
4) Your team decides to buy your dodge ball coach a gift at the end of the season. Your coach likes word games (Scrabble, Boggle, or Wheel of Fortune), old video games (PacMan or Ms. PacMan), and candy (M\&M's or Snickers), so you decide to all chip in and buy one of each. Draw a tree to determine how many different gifts your team could give your coach and then use the Fundamental Principle of Counting to verify.
5) How many different sequences of heads and tails are possible if a coin is flipped 8 times?
6) From four sets of mixed doubles bridge tournament winning partners, in how many ways can a new pair of mixed doubles bridge partners consisting of a man and a woman be chosen if the original partners can't play together?
7) In the locker room after practice, you can't remember your combination. If it consists of 5 single digit numbers ( $0-9$ ), how many different combinations are possible for you to try?
8) A sporting goods store has fourteen lines of snow skis, seven types of bindings, nine types of boots, and three types of poles. Assuming that all items are compatible with each other, how many different complete ski equipment packages are available?
9) You have 9 different skateboard stickers all of which would fit on your skateboard at once. How many different ways could you decorate your skateboard- using all of the stickers, some of the stickers, or none?
10) On their way to a Pacers game, how many ways can a family of five be seated in a car if either two or three people sit in the front seat and only two members of the family can drive?
11) A high school offers 6 varsity sports in the fall, 3 in the winter, and 5 in the spring. How many different 3 sport athletes could there be at this high school?
12) How many four-digit numbers (meaning they don't start with a 0 ) are there that do not contain a 5 ?
13) A trifecta in horse racing consists of choosing the exact order of the first three horses across the finish line. If there are 6 horses in the race, how many trifectas are possible, assuming there are no ties?

## Group \#11

## Poker Chips Permutations and Combinations



Each group has a bag of different colored poker chips. For each question below rearrange the chips to determine the answers. Use $\mathbf{B}$ for blue, $\mathbf{R}$ for red, $\mathbf{W}$ for white, and $\mathbf{M}$ for maroon.
11.1) Use three different colored chips from the bag. List the outcome(s) and the total number of ways for each.
a) How many ways are there to grab one chip?

Total number: $\qquad$
b) How many ways are there to grab a $1^{\text {st }}$ chip and then a $2^{\text {nd }}$ chip without replacement?

Total number: $\qquad$
c) How many ways are there to grab a $1^{\text {st }}$ chip, then a $2^{\text {nd }}$, and then a $3^{\text {rd }}$ without replacement?

Total number: $\qquad$
d) How many ways are there to grab 2 of the 3 chips at once?

Total number: $\qquad$
e) How many ways are there to grab all 3 chips at once?

Total number: $\qquad$
f) How many ways are there to put the chips in a circle?

Total number: $\qquad$
11.2) Now use four different colored chips. List the outcome(s) and the total number of ways for each.
a) How many ways are there to grab one chip?

Total number: $\qquad$
b) How many ways are there to grab a $1^{\text {st }}$ chip and then a $2^{\text {nd }}$ chip without replacement?

Total number: $\qquad$
c) How many ways are there to grab a $1^{\text {st }}$ chip, then a $2^{\text {nd }}$ chip, and then a $3^{\text {rd }}$ chip without replacement?

Total number: $\qquad$
d) How many ways are there to order the four chips in a row? You don't have to list them all, but using c) tell how you could easily get all of the ways.

Total number: $\qquad$
e) How many ways are there to choose 2 of the 4 chips at once?

Total number: $\qquad$
f) How many ways are there to choose 3 of the 4 chips at once?

Total number: $\qquad$
g) How many ways are there to choose all 4 chips at once?

Total number: $\qquad$
h) How many ways are there to put the chips in a circle?
$\qquad$
11.3) For this last part, take both blue chips (B1 and B2) and all three white chips (W1, W2, and W3) out, so you'll have 5 chips total. List the outcome(s) and the total number for each. For all of these order doesn't matter; you grab the chips at the same time.
a) How many ways are there to grab one chip?

Total number: $\qquad$
b) How many ways are there to choose 2 blue chips?

Total number: $\qquad$
c) How many ways are there to choose 2 white chips?

Total number: $\qquad$
d) How many ways are there to choose 1 blue and 1 white?

Total number: $\qquad$
e) How many ways are there to choose 2 chips?

Total number: $\qquad$
f) How many ways are there to choose 2 blue and 1 white?

Total number: $\qquad$
g) How many ways are there to choose 1 blue and 2 white?

Total number: $\qquad$
h) How many ways are there to choose 3 chips?

Total number: $\qquad$
i) How many ways are there to choose 4 chips?

Total number: $\qquad$
j) How many ways are there to choose 5 chips?

Total number: $\qquad$

Here are a few problems referring back to Bulls and Cows from Group 2 (remember, n-digit codes that you had to guess).
11.4) a) How many different 3 digit codes are there if
i) repetition is not allowed?
ii) repetition is allowed?
iii) the code number must be even and repetition is not allowed?
b) How many different 4 digit codes are there if
i) repetition is not allowed?
ii) repetition is allowed?
iii) the code number must be even and repetition is allowed?
c) How about 5 digit codes if
i) repetition is not allowed? $\quad$ ii) repetition is allowed?
iii) the code number must start with an 8 , end with a 4 , and repetition is not allowed?

## PERMUTATIONS AND COMBINATIONS NOTES

Factorials

$$
\mathrm{n}!=n(\mathrm{n}-1)(\mathrm{n}-2) \ldots(3)(2)(1)
$$

$0!=\quad 1!=\quad 3!=\quad 4!=\quad 5!=\quad 6!=$
$7!=$
$8!=$
$9!=$
$10!=$

Permutations: ordered arrangements of objects (distinguishable) ORDER MATTERS!!!

$$
\mathrm{nPr}=\mathrm{P}(\mathrm{n}, \mathrm{r})=
$$

Ex7. If a family of four decides to bowl on their Wii, how many orders are there for the family to bowl?

Ex8. a) If there are 11 women in a tennis tournament, how many ways are there for them to finish the tournament in $1^{\text {st }}, 2^{\text {nd }}$, and $3^{\text {rd }}$ ?

What if there were 100 women in the tournament and we wanted to know how many ways for them to finish in $1^{\text {st }}-10^{\text {th }}$ place?

Ex9. Eleven people, consisting of 5 men, 2 women, and 4 kids, are going to a Butler basketball game and have 11 seats in a row. How many ways are there to seat them if:
a) no restrictions?
b) the grown-ups sit together and the kids sit together?
c) the women sit together, the men sit together, and the kids sit together?

Circular Permutations: to seat n people in a circle, there are (n-1)! ways
Ex10. If Brad invites 4 of his friends over to play poker one Friday night, how many ways are for the 5 to sit at Brad's round poker table?

Combinations: Order does NOT matter $\quad \mathrm{nCr}=\mathrm{C}(\mathrm{n}, \mathrm{r})=\quad$ use nCr key on calculator

Ex8. Of the 11 women in a tennis tournament, how many ways are there for the players to finish in the top three?

Ex9. Let's say a group of 11 people (consisting of 5 men, 2 women, and 4 kids) only have 6 tickets for an Indiana Ice hockey game. How many ways are there for them to choose who gets to go to the game if:
a) no restrictions?
b) four grownups and two kids get to go?
c) two men, one woman, and three kids get to go?
d) at least 2 kids get to go?

## HOMEWORK PROBLEMS

1) Using a standard deck of 52 cards, how many different seven card hands are possible?
2) How many ways can 10 different countries parade in at the Olympic games?
3) How many ways can twelve people be divided into hockey teams of six players each?
4) From 5 Sorry players, 4 Chutes and Ladders players, and 3 Uno players ( 12 players total), a committee of 6 is to be chosen. In how many ways can this be done if:
a) there are no restrictions?
b) the committee must include 3 Sorry players, 2 Chutes and Ladders players, and 1 Uno player?
c) exactly 2 members of the committee are Uno players?
d) at least three are Sorry players?
5) In a chess club consisting of sixteen members, how many ways are there of choosing a committee of six if
a) no restrictions? $\quad$ b) the president must be on the committee and another member is not able to serve?
6) In how many ways can three bowlers be selected out of fifteen if:
a) one of the bowlers is included in every selection?
b) two of the bowlers are to be excluded from every selection?
c) one is always included and two are always excluded?
7) How many 14 -card hands having exactly 10 spades can be dealt?
8) From a standard deck, how many five-card hands containing only hearts and diamonds can be dealt?
9) How many 5-card hands dealt from a standard deck of cards will have 3 aces and 2 kings?
10) From a league of 7 track and field teams, how many ways can they end the season:
a) in $1^{\text {st }}, 2^{\text {nd }}$, and $3^{\text {rd }}$ place?
b) in the top three?
11) How many ways can 4 member of a relay race team be positioned around a track if you
a) need Janine to start and Charlotte to finish?
b) are not worried about who starts and ends because they're just practicing and will keep going around and around?
12) In how many ways can three girls and two boys on a co-ed volleyball team be introduced before a game, given the following conditions?
a) there are no restrictions
b) the boys go before the girls
c) girls together and boys together
d) the boys and girls alternate
13) A $5 / 36$ lottery requires choosing five of the numbers 1 through 36 . How many different lottery tickets can you choose? (Order is not important and numbers don't repeat)
14) Now figure out how many different lottery tickets you can choose in a $6 / 53$ lottery (given the same directions as above).
15) Which lottery would be easier to win, a $5 / 36$ or a $6 / 53$ ? Why?
16) A softball league has 14 teams. If every team must play every other team once in the first round of league play, how many games must be scheduled?
17) Two hundred people buy raffle tickets. Three winning tickets will be drawn at random.
a) If first prize is $\$ 100$, second is $\$ 50$, and third is $\$ 20$, in how many different ways can the prizes be awarded?
b) If each prize is $\$ 50$, in how many ways can the prizes be awarded?
18) There are 4 fans who will sit in 6 seats in a row at a Colts game. In how many ways can the fans be seated?
19) You have 4 dogs, 2 cats, and 3 guinea pigs and they all need to go to the vet. How many ways are there to make appointments for them with
a) no restrictions?
b) the dogs go first, the cats go next, and the guinea pigs go last?
c) the dogs have consecutive appoints, the cats have consecutive appointments and the guinea pigs have consecutive appointments but in any order?
20) How many different 9 digit social security numbers are there with exactly seven 0 's?

## Group \#12

## Is order important? II Crunching the Numbers



We already discussed in groups if order was important, so use this knowledge and what we know about combinatorics, permutations and combinations to determine the answers to the following questions. MAKE SURE TO SHOW YOUR WORK, NOT JUST GIVE AN ANSWER (use factorials and nCr , etc).
12.1) A university has three table tennis scholarships each worth $\$ 1000$. How many ways are there to award these after they've narrowed the candidates to 6 ping pong players?
12.2) This same university has three badminton scholarships, one worth $\$ 3000$, one worth $\$ 2000$, and one worth $\$ 1000$. If the field of qualified badminton candidates is also narrowed to 6 , how many ways are there to give the scholarships?
12.3) How many ways are there for 8 teams in a soccer league to end the season in $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ place?
12.4) How many ways are there for 7 teams in a football league to end the season in the top three?
12.5) After being dealt five cards, how many different ways can a player arrange the cards in their hand?
12.6) How many different foursomes can be formed from among 6 golfers?
12.7) How many different finishes can occur in an 8 -horse race (excluding the possibility of ties)?
12.8) How many different 9 player batting line ups are possible on a softball team with a 13 -player roster?
12.9) How many different ways are there for a soccer team of 11 to choose 3 players to be team captains?
12.10) How many different 5 card poker hands are there?
12.11) How many different solutions are there for the game of Clue if the game has 9 rooms, 6 weapons, and 6 suspects?
12.12) How many different outcomes are there when you flip a penny three times in a row?
12.13) How many different outcomes are then when you flip three identical nickels at the same time?
12.14) How many different blackjack hands are there (first card dealt down, second dealt face up)?
12.15) In how many ways can 7 go-karts be situated on a circular track during a race, assuming that no two are exactly side by side?
12.16) How many ways are there to seat 4 people around a table to play bridge?
12.17) A 3 reel slot machine has 10 different symbols on each reel (including one JACKPOT symbol). How many different outcomes are there for this slot machine?
12.18) How many ways are there to grab 3 marbles out of a bag with 6 red marbles, 5 green marbles, and 2 blue marbles?
12.19) How many ways are there to grab 3 marbles out of that same bag so that the first is red, the second is green, and the third is blue?
12.20) How many different 3 piece ski outfits could you come up with if you have 2 jackets, 5 hats, and 4 pairs of gloves?
12.21) There are 100 squares on a Battleship game board. How many ways could you start off the game during your first three shots?
12.22) Uno is played with a deck of 108 Uno cards and everyone is dealt 7 cards. How many different Uno hands are there?
12.23) A family of 4 is playing on the game show Family Feud. How many different ways could they sit in a row to be asked questions?
12.24) Sixteen marching bands have volunteered to perform at the Lucas Oil Stadium during half time of a Colts game, but there in only enough space for 10 of the bands to be on the field. How many ways are possible for the bands to be chosen?
12.25) A coed softball team consists of 7 women and 6 men. How many different ways can 2 captains be picked if there must be one man and one woman?
12.26) A hockey league has 15 teams. If every team must play every other team once in the first round of league play, how many games must be scheduled?
12.27) How many different "words" (can be nonsense words) can be formed using all of the letters in BIKE?
12.28) Given a group of 20 cheerleaders, 15 females and 5 males, how many ways can a group of 10 be picked to perform in a competition if there must be 7 females and 3 males?
12.29) If you are rolling a single die, how many ways are there to roll exactly two 4 's in three rolls?
12.30) There are 10 marbles in a bag, 5 green, 3 blue, and 2 red and you want to grab 3 of them, one at a time without replacement. How many ways are there to reach in and grab a green, and then a blue and then another green?

You are the coach of a baseball team called the Strikers which has 12 kids ( 7 boys and 5 girls) on it. Make sure you show work on these problems to show how you're getting your answer.
12.31) Positions
a) How many different ways are there to rest three of the players when your team is out in the field?
b) After you've chosen your three who will rest, how many ways are there to assign your remaining nine players to either the outfield (3) or the infield (6) where their actual positions are not important, only infield or outfield?
c) How many ways can you assign the three selected for outfield to right/left/ or center?
d) And the six in the infield $\left(1^{\text {st }}, 2^{\text {nd }}, 3^{\text {rd }}\right.$, shortstop, catcher, and pitcher $)$ ?
e) The league has a rule that at least 4 girls must be playing in the field. How many ways can you assign nine of the players from the 12 to play the field (no positions)?
f) Let's say you've chosen your nine to play on the field consisting of all 5 girls. How many ways can you assign them to the outfield or infield if 3 of the girls must be in the infield?
12.32) Batting Order
a) How many different nine person line ups are there for you to choose from?
b) You want your fastest base runner to lead off and your star hitter to be in the clean-up position ( $\left.4^{\text {th }}\right)$, how many line ups are there?
c) What if you have in mind the top 4 batters whom you would like to be at the top of the batting order (but not their order) and then fill in the remaining 5 spots?

## Group \#13

## Poker Hands and Combinations



When most people think of poker, they think of straight poker which is played with each player dealt 5 cards. In this group project we will be using combinatorics to count the number of different types of 5 card poker hands. Note that order doesn't matter; a poker hand is the same no matter how you were dealt your cards.
13.1) How many different 5 card poker hands are there?

How many 5 card poker hands are there with the following properties:
13.2) all black?
13.4) all hearts?
13.3) only clubs and diamonds?
13.5) 3 black and 2 red?
13.8) 2 spades and 3 hearts?
13.10) exactly 2 hearts?
13.12) at least 4 black?
13.14) at least 4 clubs?
13.16) 4 tens and 1 ace?
13.18) at most 1 six?
13.11) exactly 4 spades?
13.13) at most 2 hearts?
13.15) 3 jacks and 2 eights?
13.20 ) all face cards?
13.21) no aces?
13.22) at least one ace? 13.23) exactly 2 face cards?
13.24) four of a kind (4 of one rank and one card of a different rank)?

## Group \#14

## Volleyball Tournaments



You are on a volleyball team called the Spikers and your team plays in many tournaments, including single elimination, double elimination, best of three, and best of five. In this group, you will determine how many games will be played in each type of tournament with a given number of teams.
14.1) Your team is entered in a single elimination tournament (meaning one loss and your team is out). State how many total games will be played during the tournament.
a) 3 other teams ( 4 teams total)
b) 7 other teams ( 8 teams total)
c) 11 other teams ( 12 teams total)
d) If there are N teams in a single-elimination tournament, how many games would need to be played?
14.2) You've entered your team in a double elimination tournament which means you get 2 losses before your team is out. a) If there is only one other team involved (all the other teams were sick with the flu), how many games could be played with 2 teams total? (There are two answers.)
b) If there are 2 other teams, how many possible games with 3 teams total? Imagine if the winning team goes undefeated or if they have one loss (there will be a different answer for each scenario).
c) How about 3 other teams so 4 teams total?
d) Make a guess as to how many possible games could be played if there are N teams involved in a double elimination tournament. (Again, there will be two possibilities.)
14.3) Let's say the Spikers make it to the league championship which is a Best of Three series (first team to win 2 games, wins). They will be playing the Diggers. Note that there will either be 2 or 3 games played.
a)Draw a tree diagram to show how the series could end. Each branch will represent the team who wins each game. Start with game 1 either the Spikers (S) will win or the Diggers (D), then the next game, etc. Stop once a team has won 2 games.

b) List the possible outcomes for this best of three series by following each branch to the end, noting who won each of the games.
14.4) Now let's say the Spikers and Diggers are playing a Best of Five series (first team to win 3 games wins the series, so there could be 3 , 4 , or 5 games played).
a) Draw a tree diagram to represent the possibilities. Hint: This tree will be much bigger with 20 possible outcomes.
b) List the possible outcomes for this Best of Five series.
14.5) The Spikers have entered a round robin tournament which is one where every team plays every other team and then their records are compared with several tie breakers for the same win/loss ratio. (A lot of tournaments start with a round robin portion and then move on to single elimination.)
a) How many games would be played with 2 other teams ( 3 total)?
b) How many games with 3 other teams ( 4 total)?
c) How about 4 others teams ( 5 total)?
d) Can you come up with a formula to determine the number of games for N teams in a round robin tournament?
14.6) Then look on the internet and find 2 examples of leagues that use the formats and how they use them (for example Major League Baseball uses a best of seven series for the World Series- you can't use that example now!).
a) Best of three series: Two teams play 2 or 3 games; whoever wins 2 games first wins the series.
$1^{\text {st }}$ example:
$2^{\text {nd }}$ example:
b) Best of five series: Two teams play 3, 4, or 5 games; whoever wins 3 games first wins the series.
$1^{\text {st }}$ example:
$2^{\text {nd }}$ example:
c) Best of seven series: Two teams play 4, 5, 6 , or 7 games; first to win four games wins the series.

List 2 possible outcomes of each where $4,5,6$, and 7 games had to be played to determine the winner:

4 games:

5 games:

6 games:
$1^{\text {st }}$ example:
$2^{\text {nd }}$ example:
7 games:
14.7) How many games are played in the NCAA basketball tournament (single elimination) with 63 teams and two teams that play a "play-in" game to join the brackets?
14.8)

| position | $1^{\text {st }}$ | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ |
| :---: | :--- | :--- | :--- | :--- |
| first name |  |  |  |  |
| surname |  |  |  |  |
| cake |  |  |  |  |

The

NCAA tournament has changed to allow 68 teams in; how many games are played now?
14.9) How many games would there be if the NCAA basketball tourney were a double elimination tournament with 68 teams?

## Test \#1 Review Problems

1) Determine the surnames of four people, their position in a cake-bake competition and what cake they baked.

The first names are: James, Ben, Vicky and Nigel
The surnames are: Jones, Stevens, Andrews and Best
Positions are: 1st, 2nd, 3rd and 4th
Cakes baked are: chocolate cake, cheese cake, fruit cake and sponge cake.

1. James Best beat Vicky by two places.
2. Ben's fruit cake beat Mrs. Stevens' son, who came in 3rd.
3. The sponge cake, despite being a bit bland, got in the top three.
4. The judges obviously had a sweet tooth, as the chocolate cake came in 2nd place.
5. Jones's mother cried as she watched her son take 1st prize.

What are the full names of the top four, what position did they come and what cake did they bake?
2) Use set notation to describe the shaded region.

3) Given the sets: $U=\{1,2,3,4,5,6,7\}, A=\{3,4,5,6\}, B=\{1,2\}, C=\{1,3,5,7\}$, and $D=\{7\}$, determine the following using proper set notation:
a) TRUE or FALSE $A^{C}=B \cup D$
h) TRUE or FALSE $A=\{4,6,3,5\}$
b) list the subsets of $D=$ $\qquad$ i) $\{2\}_{\ldots} \quad B$ fill in with $\in$ or $\subseteq$
c) TRUE or FALSE $\varnothing \subseteq D$
j) TRUE or FALSE $\varnothing \not \subset U$
d) $(B \cup C) \cap A=$ $\qquad$ k) TRUE or FALSE $\varnothing=\{\varnothing\}$
e) the number of subsets of $C=$ $\qquad$ 1) TRUE or FALSE $B \subset C$
f) $U^{c}=$ $\qquad$ m) $(B \cup D)^{C} \cap C=$ $\qquad$
g) $A \cup C=$ $\qquad$ n) $n(A \cup C)=$ $\qquad$
4) Given $n(L)=570, n(L \cup M)=700, n(L \cap M)=60$, find $n(M)$.
5) The management of a hotel conducted a survey and found that of the 2560 guests who were surveyed,

1780 tip the wait staff 1200 tip the luggage handlers 750 tip the wait staff and the luggage handlers 830 tip the maids 200 tip all three services 80 tip the maids and the luggage handlers but not the wait staff

Fill in the VENN DIAGRAM completely and answer the following: How many of the guests tip:

a) exactly two of the services?
b) the wait staff or the maids?
c) only one of the three services?
d) do not tip any of these services?
6) Shade the region on the Venn diagram that represents $\left(A \cap B^{C}\right) \cup\left(A^{C} \cap C\right)$.

7) Fill in the sudoku.

|  | 3 | 7 | 9 | 6 |  | 4 | 8 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 4 | 5 |  | 3 | 8 | 9 | 1 | 7 |
| 1 | 8 |  | 5 | 4 | 7 | 6 |  | 3 |
|  | 7 | 1 | 4 | 8 | 9 | 2 | 5 | 6 |
| 9 | 5 |  | 6 | 2 |  | 7 |  | 1 |
| 4 |  | 6 | 1 |  | 5 | 8 | 3 | 9 |
| 7 | 1 | 2 | 8 |  | 6 | 3 | 9 | 4 |
| 5 | 6 | 4 | 3 | 9 | 2 |  | 7 |  |
| 8 |  | 3 |  | 1 | 4 | 5 | 6 | 2 |

8) Nine individuals - $Z$,

Y, X, W, V, U, T, S and R - are the only candidates, who can serve on three committees-- A, B and C, and each candidate should serve on exactly one of the committees.

Committee A should consist of exactly one member more than committee B.
It is possible that there are no members of committee $C$, but $A$ and $B$ must have at least one member each.
$\mathrm{Z}, \mathrm{Y}$ and X can NOT serve on committee A .
$\mathrm{W}, \mathrm{V}$ and U can NOT serve on committee B.
$\mathrm{T}, \mathrm{S}$ and R can NOT serve on committee C .
a) If T and Z are the only ones serving on committee B , how many of the 9 individuals should serve on committee C ?
b) Of the nine individuals, the largest number that can serve together on committee C is
c) If $T, S$ and $X$ are the only individuals serving on committee $B$, who should be on committee $C$ ?
9) There are 11 people on a committee. How many ways are there for them to:
a) elect a president, vice president, and secretary?
b) choose a subcommittee of 3 ?
10) How many five card hands dealt from standard deck of cards will have:
a) all spades?
b) 3 tens and 2 face?
c) at least 4 of the cards are black?
11) From 5 physicists, 4 chemists, and 3 mathematicians a committee of 6 is to be chosen. How many ways can this be done if:
a) there are no restrictions?
b) committee must include 3 physicists, 2 chemists, and 1 mathematician?
c) exactly 2 members of the committee are mathematicians? d) at least three are physicists?
12) A cafeteria offers 2 choices of meat, 4 choices of veggies, 3 choices of drink, and 6 choices of fruit. How many 4 -item lunches are available?
13) How many three digit numbers are possible from the digits $1,2,4,6,7,8$ if repetition:
a) is allowed?
b) is not allowed?
c) is not allowed and the number must be even?
14) In the game Bulls and Cows, a BULL = correct digit in the right position and a COW $=$ correct digit in the wrong position. Determine the code for the example.

| Guess 1:123 | $\rightarrow$ | 1 bull and 0 cows |
| :--- | :--- | :--- |
| Guess 2: 456 | $\rightarrow$ | 0 bulls and 1 cow |
| Guess 3: 789 | $\rightarrow$ | 0 bulls and 0 cow |
| Guess 4:140 | $\rightarrow$ | 0 bulls and 1 cow |
| Guess 5: 025 | $\rightarrow$ | 0 bulls and 2 cows |

Code: $\qquad$
15) How many games would be played in a tournament with 14 teams which is: a) single elimination?
16) Given a standard deck of cards, find $n(A)$ for each set:
a) $\mathrm{A}=\{$ clubs and red $\}$
b) $\mathrm{A}=\{$ nines or spades $\}$
c) $\mathrm{A}=\{$ twos or hearts, but not fours $\}$

