MATHEMATICS WITHOUT CALCULATIONS

IT'S A BEAUTIFUL THING!

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MATHEMATICS WITHOUT CALCULATIONS IT'S A BEAUTIFUL THING!

- Seminar style class for freshmen non-math majors - about 15 students.

- Meets twice per week for 75 minutes per meeting.

- Purpose of the freshman seminars is to teach college level writing and speaking in the context of a discipline.

- Textbook: "The Heart of Mathematics, An Invitation to Effective Thinking" Burger and Starbird

- Supplement: "Harbrace Essentials with Resources for Writing in the Disciplines" by Cheryl Glenn and Loretta Gray. Publisher: Wadsworth Cengage Learning

CLASS FORMAT

- End of the previous class: Students are assigned a short reading and asked to write a one to two page reflection (with guidelines given).

- Beginning of Class: Discussion. Everyone is asked to state something interesting found in the previous night's readings.

- An interactive lecture is based off this discussion. (The students are writing my lecture notes!)

- Groupwork: Mindscapes are assigned to reinforce the concepts. Class discussion follows.

- End of class: The next reading and writing assignment is assigned. Necessary background information is given.

SAMPLE NIGHTLY WRITING ASSIGNMENTS

- What is math?

NATURAL NUMBERS

- Decribe the Fibonacci numbers, their relationship to the golden ratio, and their applications to art and nature.

- Explain why there are infinitely many primes.

- Explain why the square root of a prime number is irrational.

CARDINALITY

- Describe the difference between rational and irrational numbers and explain their roles in the context of the real numbers. Are there more rational or irrational numbers? Why?

- Describe the ping pong ball conundrum and explain how this illustrates the difference between the finite and the infinite.

- Explain why the cardinalities are the following sets are all equal: the natural numbers, the natural numbers without "1", the integers, the rational numbers.

- Explain why the cardinality of the real numbers is greater than the cardinality of the natural numbers.

- Expand on the idea of the cardinality of the real numbers being greater than the cardinality of the natural numbers to show that there are infinitely many levels of infinity. Use the idea of the "Mystery Set".

THREE DIMENSIONAL SOLIDS

- Explain why for any platonic solid that $V - E + F = 2$.
- Explain why there are only five regular platonic solids.

FOURTH DIMENSION AND FRACTIONAL DIMENSIONS

• Describe the fourth dimension in terms of (1) extending lower dimensions to higher ones and (2) seeing inside objects of lower dimensions.

• Define the idea of dimension in terms of multiple copies of an object. Use the definition to explain why the Koch Curve and Menger Sponge are of fractional dimension.

GRAPH THEORY

• Using the fact that $V - E + F = 2$ for any planar graph, explain why $E \leq 3V - 6$. Use this to show that the "Five-Station Model Train Puzzle" is not possible.

• Discuss the idea of vertex coloring and relate it to the Six-Color Theorem.

MIDTERM

• Taken in class, but students are given the questions well in advance so that they can prepare.

• Three students each roll a die at the beginning of the midterm to determine the three questions that constitute the midterm. Students are asked to do any two of the three midterm questions chosen.

1. Fibonacci Numbers
(a) Explain how the Fibonacci numbers are constructed and list the first ten Fibonacci Numbers.
(b) Use the ratio of consecutive Fibonacci numbers to derive the golden ratio.

2. Prime Numbers
(a) Prove that there are infinitely many prime numbers.
(b) Given any number, show that there exists a run of that many consecutive numbers, none of which is a prime number.

3. Real Numbers
(a) Given any two real numbers explain why there are infinitely many real numbers between them.
(b) Given any real number, explain why there is no next real number.
(c) Is $0.\overline{9}$ equal to 1 or just very close to 1? Explain your reasoning.

4. Cardinality of Rationals and Reals
(a) Show that the set of rational numbers has the same cardinality as the set of natural numbers.
(b) Show that the set of real numbers has a greater cardinality than the set of natural numbers.

5. Infinity
(a) Show that for any infinite set S, the power set of S has a greater cardinality than the cardinality of S.
(b) Use part (a) to explain why there are infinitely many levels of infinity. Be sure to discuss the Continuum Hypothesis as well.

6. Proving the cardinality of $\mathcal{P}(\mathcal{N})$ equals the cardinality of the set of Real Numbers
(a) Show that the interval $(0, 1)$ has the same cardinality as the cardinality of the set of real numbers.
(b) Show that the intervals $(0, 1)$ and $[0, 1]$ have the same cardinality.
(c) Using base-2 decimals, explain why the cardinality of $\mathcal{P}(\mathcal{N})$ (the power set of the set of natural numbers) has the same cardinality as $[0, 1]$.

FINAL EXAM

1. What is math?

2. The three questions from the midterm that were not chosen by the roll of the die appear on the Final Exam. A roll of a die will determine which midterm question is done on the final exam.

3. Five questions from topics covered since the midterm appear on the final. Students must do two of these questions. A roll of the die determines one question. Students choose the other question.

• Note that all questions are given in advance. Students may hand in question 1 ahead of time. Students are allowed to bring a note card for the final exam.

3A: The fourth dimension
(a) Explain the concept of the fourth dimension using the following ideas: dragging, seeing inside objects, and untying knots.
(b) Draw a picture of a four-dimensional cube and explain how you obtained this from the three-dimensional cube.

3B: Fractional dimensions
(a) Using the concept of the scaling factor, explain what is meant by a fractional dimension.
(b) Illustrate the concept of a fractional dimension by showing how the dimension of two of the following three objects are calculated: Koch curve, Sierpinsky Triangle, and Menger's Sponge.

3C: Five platonic solids
(a) Prove that for any plane graph that $V - E + F = 2$
(b) Recalling that $F = 2E/s$ and $V = 2E/c$ where s is the number of edges enclosing each face and c is the number of edges emanating out of each vertex, prove that there are exactly five regular platonic solids.

3D: Planarity

(a) Using the fact that a face must be bounded by at least 3 edges, prove that $E \leq 3V - 6$ for all planar graphs.

(b) Suppose you have five houses and you want each pair of houses to be connected by a wire. Using part (a), show that it is impossible to do this without a pair of wires crossing. (Hint: Think about the graph K_5.)

3E: The Six Color Theorem

(a) Using the fact that $E \leq 3V - 6$, show that a planar graph must have a vertex of degree 5 or less.

(b) Prove the six color theorem.

RESEARCH PAPERS AND PRESENTATIONS

- 6-7 pages, APA Style

- Minimum of four sources, at least one book and one internet source. The textbook may be used, but it does not count as one of the four sources.

- One class meeting early in the semester takes place in the library with a librarian giving a presentation on library resources.

- Presentations are 10-12 minutes in length

- Presentations must be uploaded onto Blackboard

- Timeline for Fall Semester:

◦ September 19th Outline Due

◦ October 8th Rough Draft Due

◦ October 17th Peer Comments Due

◦ October 29th Individual Consultations

◦ November 7th PowerPoint Presentations Due

◦ November 14th Final Draft of Paper Due

◦ November 14th Presentations begin

RESEARCH TOPICS

- Combinatorics (combinations, permutations, Pascals Triangle, basic probability)

- Conic Sections (eccentricity, slicing the cone, explanation of graphs)

- Continued Fractions (Golden Ratio, pi)

- Game Theory (mathematics of games, puzzles)

- Geometry (history, proof of Pythagorean Theorem, Euclidean vs. non-Euclidean geometry)

- Graph Theory (basic definitions and types of graphs, Eulerian circuits, Hamiltonian circuits, trees, minimum spanning tree, algorithms such as Kruskal's)

- History of Mathematics
- Leonard Euler (his contributions to mathematics)
- Linear Algebra (matrix theory)
- Modular Arithmetic (congruence mod n, applications, public key encryption)
- Number Systems (Historical numeration systems, Hindu-Arabic Systems, Binary and hexadecimal numbers, conversion between number bases)
- Number Theory (prime numbers, gcd, lcm, Euclidean algorithm, Diophantine equations, Fermat Last Theorem)

- Statistics (probability, normal curve, confidence intervals, standard deviation)

- Three-Dimensional Solids (polyhedra, platonic solids, Archimedean solids)

- Trigonometry (history, development of sine and cosine, unit circle, right triangle trigonometry)

- Unsolved / Recently Solved Problems in Mathematics (Four Color Theorem, Fermat's Last Theorem, Goldbach's Conjecture, Twin Prime Conjecture, much more!)

- Voting Theory (Mathematics of voting)