Math as a Creative Art

Reflections on an honors proofs class for liberal arts majors

Nora Devlin, Pat Devlin

Rutgers

August 5, 2016

Outline

Outline

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote.

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

- ". . . I suck at a math." [self view]

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

- ". . . I suck at a math." [self view]
- ". . . I hate math." [opinion of mathematics]

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

- ". . . I suck at a math." [self view]
- ". . . I hate math." [opinion of mathematics]
- ". . . Wow, you must be really smart." [view of mathematicians]

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

- ". . . I suck at a math." [self view]
- ". . . I hate math." [opinion of mathematics]
- ". . . Wow, you must be really smart." [view of mathematicians]
- ". . . Did you discover any new numbers?" [what is math]

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

- ". . . I suck at a math." [self view]
- ". . . I hate math." [opinion of mathematics]
- ". . . Wow, you must be really smart." [view of mathematicians]
- ". . . Did you discover any new numbers?" [what is math]
- ". . . I think the common core . . ." [math in society]

Course overview

- New course designed and implemented at Rutgers in fall 2014
- Honors students in liberal arts
- One semester course
- Twice weekly meetings, 80 minute sessions

A driving motivation for the course:

Complete the following quote. "Oh! You're a mathematician? . . ."

- ". . . I suck at a math." [self view]
- ". . . I hate math." [opinion of mathematics]
- ". . . Wow, you must be really smart." [view of mathematicians]
- ". . . Did you discover any new numbers?" [what is math]
- ". . . I think the common core . . ." [math in society]
- ". . . Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

- "...I suck at a math." [self view]
- "...I hate math." [opinion of mathematics]
- "...Wow, you must be really smart." [view of mathematicians]
- "...Did you discover any new numbers?" [what is math]
- "...I think the common core ..." [math in society]
- "...Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

Course goals

- "...I suck at a math." [self view]
- "...I hate math." [opinion of mathematics]
- "...Wow, you must be really smart." [view of mathematicians]
- "...Did you discover any new numbers?" [what is math]
- "...I think the common core ..." [math in society]
- "...Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

Course goals

Present "math as a creative art" so that

- "...I suck at a math." [self view]
- "...I hate math." [opinion of mathematics]
- "...Wow, you must be really smart." [view of mathematicians]
- "...Did you discover any new numbers?" [what is math]
- "...I think the common core ..." [math in society]
- "...Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

Course goals

Present "math as a creative art" so that

- students have first-hand experience with math (i.e., proofs)

- "...I suck at a math." [self view]
- "...I hate math." [opinion of mathematics]
- "...Wow, you must be really smart." [view of mathematicians]
- "...Did you discover any new numbers?" [what is math]
- "...I think the common core ..." [math in society]
- "...Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

Course goals

Present "math as a creative art" so that

- students have first-hand experience with math (i.e., proofs)
- students discover beautiful mathematical results and proofs

- "...I suck at a math." [self view]
- "...I hate math." [opinion of mathematics]
- "...Wow, you must be really smart." [view of mathematicians]
- "...Did you discover any new numbers?" [what is math]
- "...I think the common core ..." [math in society]
- "...Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

Course goals

Present "math as a creative art" so that

- students have first-hand experience with math (i.e., proofs)
- students discover beautiful mathematical results and proofs
- students recognize and investigate societal assumptions about math and mathematicians

- "...I suck at a math." [self view]
- "...I hate math." [opinion of mathematics]
- "...Wow, you must be really smart." [view of mathematicians]
- "...Did you discover any new numbers?" [what is math]
- "...I think the common core ..." [math in society]
- "...Ok, why does (doesn't) $0.9999\ldots = 1$?" [content of math]

Course goals

Present "math as a creative art" so that

- students have first-hand experience with math (i.e., proofs)
- students discover beautiful mathematical results and proofs
- students recognize and investigate societal assumptions about math and mathematicians
- (ideally!) students change their opinions about math and their relation to it

Outline

Structure

In class:

- In-class activities and student-driven exploration of topics
- Ideal was for instructor to be as 'out of the way' as possible
- Purposely very flexible about what content is discussed
- Guest facilitators (Neil Sloane, Rebecca Coulson, Kellen Meyers, et cetera)

Structure

In class:

- In-class activities and student-driven exploration of topics
- Ideal was for instructor to be as 'out of the way' as possible
- Purposely very flexible about what content is discussed
- Guest facilitators (Neil Sloane, Rebecca Coulson, Kellen Meyers, et cetera)

Graded work:

- Weekly readings and written reflections
- Final project
- Exams [in retrospect, these were unnecessary]

Content

Example content:

- Combinatorial games (subtraction games, nim, puzzles)
- Graph theory (Ramsey theory, colorings, Eulerian graphs)
- Number theory (primes, sequences, $\sqrt{2} \notin \mathbb{Q}$)
- Infinity (Zeno's paradox, $0.\overline{9} = 1$, infinite sums, Hilbert's hotel, $|\mathbb{Z}| = |\mathbb{Q}| = \aleph_0 \neq |\mathbb{R}|$)
- Beyond (topology, algorithms, formal logic, axiom of choice, Banach-Tarski, incompleteness theorems, decidability, geometric constructions)

Assignments

Example writing assignments:

- Write a reflection on *Lockhart's Lament*.
- Reflect on two recently published articles comparing math education in US with that in Japan.
- Make up a game and analyze it.
- Look up Alan Turing and describe his life, death, and intellectual contributions.
- Write a Socratic dialogue explaining why $\sqrt{2}$ is irrational.
- Talk to a family member or friend about what you learned in this course and reflect on the conversation.

Activities

Example activities:

- Bean subtraction game tournament
- Make up games and puzzles for homework and give them to each other
- Cut möbius strips in half. Try to describe any patterns.
- Try to find knots with crossing number 4 or 5 or 6
- What shapes can you draw without lifting your pencil or retracing any lines?
- Try to find a planar graph that needs k colors
- Try to find a planar graph with as many edges as possible
- Pass a ball around a circle (skip k each time). Try to notice patterns in how many people end up touching the ball.

Outline

Inclusive curriculum

"Math is all encompassing and non-exclusive. . . 'real math' is welcoming to all." — *Student (history/law major)*

- Diversity within mathematics: ethnic, racial, gender, sexual orientation, socio-economic status
- Ramanujan (self-taught, non-European, person of color)
- Alan Turing
- Maryam Mirzakhani, Sofie Germain, Vi Hart

Inclusive curriculum

"Math is all encompassing and non-exclusive. . . 'real math' is welcoming to all." — *Student (history/law major)*

- Diversity within mathematics: ethnic, racial, gender, sexual orientation, socio-economic status
- Ramanujan (self-taught, non-European, person of color)
- Alan Turing
- Maryam Mirzakhani, Sofie Germain, Vi Hart

An inclusive approach invited students to read about, respond to, and research problems both as mathematicians (proofs) and as liberal arts majors (through asking questions related to social issues, privilege, and representation). They proved they were capable scholars on multiple fronts: from graph theory to racism, sexism to game theory, Turing Tests to Turing's tragic death.

Outline

Final projects

"...students are enabled to develop their own set of achievements." [1]
Example projects:

Final projects

"...students are enabled to develop their own set of achievements." [1]
Example projects:

- Mathematics of paper folding
- "Six degrees to Theodore Roosevelt"
- "Blacks in math: where are they?"
- Sabermetrics
- "*We Solved an MTV Reality Show Weeks Before the Finale Using Simple Math:* Mathematical spoilers ahead"
- Innumeracy
- Math and magic tricks
- Riemann Zeta function: the game!
- Book of small knots (drawn beautifully)

Student change

- Quiz 0 questions, weekly reflections

Student change

- Quiz 0 questions, weekly reflections
- "Although I still cannot do calculus, I have a feeling that the logic and other mathematical concepts I studied in this class, as well as the extra knowledge I absorbed from the readings we did for homework and from the guest presenters, will be even more valuable to me in my academic career." (*history major*)

Student change

- Quiz 0 questions, weekly reflections
- "Although I still cannot do calculus, I have a feeling that the logic and other mathematical concepts I studied in this class, as well as the extra knowledge I absorbed from the readings we did for homework and from the guest presenters, will be even more valuable to me in my academic career." (*history major*)
- "I am still not a fan of math, but I can now understand why people like it. . . . [My high school] math class was the last thing anyone, especially me, wanted to talk about, but this year I find myself talking about [math] quite frequently and playing the 1,2 subtraction game with a lot of people. I think I have a more positive attitude of the subject because I took this class." (*psychology major*)

Student change

- "Not too long ago, I believed math was a cold, dry practice involving some equations, perhaps some geometric figures, and of course — numbers. But math as a 'cold, dry practice' is an unfortunate outcome of cold, dry teaching. While I have little words to articulate what this means, I, as a mathematician, find mathematical thought to be beautiful." (*communications major*)

References

Hannah Morgan and Ann-Marie Houghton.
Inclusive Curriculum Design in Higher Education: Considerations for effective practice across and within subject areas.
The Higher Education Academy, 5 2011.

More student quotes

- "I was even able to express my findings in algebraic equations which, although I am not enthusiastic about algebraic equations, made me proud. . . . I was amazed that I was able to independently find a pattern at all." (*history major*)
- "Hey, STEM is going places! It is incredibly exciting and refreshing to see a prominent mathematician like Terence Tao on *The Colbert Report*. . . . He mentioned that primes go on forever and that they are a countable infinite set (hey, I've heard that before). He also talked about the different types of primes, such as twin, cousin, and sexy primes." (*media studies major*)

More student quotes

- "I liked being able to write about what I learned rather than take home problem sets. Being able to reflect rather than reuse knowledge was a lot less stressful. Because I learned that math was more of an art and an interest than a complete necessity, I didn't feel the need to always ask, 'When am I ever going to have to know this?' I found out that math could actually be fun, and that math can also be playing games. For example, prior to this year, I had never known how to solve the river-crossing problem with the fox, goat, and hay, but after we learned about game theory I was able to solve it shortly after seeing it. I also learned some of the things I did for fun were actually related to math..." (*psychology major*)

More student quotes

- "What does it mean to be a mathematician? Does it mean one can solve any and everything, from the simplest algebraic equations to the most involved calculus problems? . . . these traits are not what make me a mathematician. Instead it is my artistic talent—my ability to be imaginative, critical, and reason. And while math and art appear to be complete opposites—one concerned with imagination and representing beauty and the other mind-numbing word problems—the same thing an 'artist' does is what I have been doing with mathematics. Moreover, because I am imaginative, creative, and see the beauty in a pattern, shape, proof, or concept in the same way artists see beauty in the world around them, I am a mathematician." (*journalism major*)

Student thoughts

- In general, quite positive
- Activities and topics chosen were interesting and engaging for most
- The course involved a good deal more work than the "100-level" would imply. (This is all but certainly the case.)
- It would have been better if more details were ironed out ahead of time and communicated to the students. (e.g., if readings and assignments were determined further in advance)