Binomial Distribution

- What is a binomial experiment?
  - What distinguishes a binomial experiment from other experiments?
- How can probability for a binomial random variable determined?

Dichotomous Responses

- Dichotomous
  - divided or dividing into two sharply distinguished parts or classifications www.thefreedictionary.com
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Dichotomous Responses

- Many experiments have dichotomous responses
  - Yes/No
  - True/False
  - Defective/Nondefective
  - Female/Male
  - On time/Not on time

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These generalize to success (getting what you want) and failure (not getting what you want).

Examples of Experiments with Dichotomous Responses

- Tossing a coin
  - Two outcomes for every toss: Heads and Tails
- True/False Test Questions
  - Two possible responses for each question: True and False
- Having a child
  - Two possible outcomes: Female and Male

Binomial Experiment

- Performed a fixed (finite) number of times
  - Each repetition of the experiment is called a trial.
- Each trial has two possible outcomes: success (getting what you want) and failure (not getting what you want)
- Each trial is independent and the probability of success is the same for each trial.

Binomial Experiment

- n trials of the experiment are performed.
- k successes and (n – k) failures among the n trials
- Probability of success: p
- Probability of failure: (1 – p).
- Each trial is independent of the next: the outcome for one trial does not affect the outcome for the next trial.

Binomial Experiment

- n trials of the experiment are performed.
- k successes and (n – k) failures among the n trials   ***Note***: $k + (n - k) = n$
- Probability of success: p
- Probability of failure: (1 – p)

Note: $p + (1 - p) = 1$

- Each trial is independent of the next: the outcome for one trial does not affect the outcome for the next trial.

Binomial Random Variable

- The number of successes in n trials of a binomial experiment is a binomial random variable.

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- The number of successes in n trials of a binomial experiment is a binomial random variable.
- Notation:
  - n is the number of trials
  - p is the probability of success
  - 1 – p is the probability of failure
  - 0 ≤ number of successes ≤ n

Is this a binomial experiment?

- A hiring committee must select two individuals to fill two positions. The committee receives ten applications, six female applicants and four male applicants, each of whom is equally qualified and deserving of the position. Suppose the selections are randomly made. Let x be the number of female applicants who are hired.

Is this a binomial experiment?

- For this experiment, each trial, selecting/hiring an applicant, affects the next.
  - First trial: P(female) = 6/10 = 3/5
  - Next trial, number of applicants decreases from 10 to 9 since one applicant has been selected/hired

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This affects P(female) for the next trial.

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Is this a binomial experiment?

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  - The birth of each child is independent
    - Gender of one child does not affect the gender of the next child
  - Two possible outcomes: female (success) and male (failure)
  - 6 trials

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- A fair coin is tossed seventeen times. Let x be the number of times the coin lands tail side up.

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- A fair coin is tossed seventeen times. Let x be the number of times the coin lands tail side up.
  - Each toss of the coin is independent
  - Two possible outcomes: Tails (success) and Heads (failure)
  - 17 trials

As usual, Tails denotes the coin landing tail side up and Heads denotes the coin landing head side up (tail side down).

Is this a binomial experiment?

- A fair coin is tossed seventeen times. Let x be the number of times the coin lands tail side up.
  - Each toss of the coin is independent
  - Two possible outcomes: Tails (success) and Heads (failure)
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- Five cards are drawn without replacement from a fair poker deck. Let x be the number of face cards drawn from the deck.
  - Since the cards are drawn without replacement from the deck, each trial affects the next.
  - The trials are not independent.

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  - Since the cards are drawn *with* replacement, each trial is independent of the next.

Is this a binomial experiment?

- Five cards are drawn *with* replacement from a fair poker deck. Let x be the number of face cards drawn from the deck.
  - 5 independent trials
  - Two possible outcomes: drawing a face card (success) and not drawing a face card (failure).

Is this a binomial experiment?

- Five cards are drawn *with* replacement from a fair poker deck. Let x be the number of face cards drawn from the deck.
  - 5 independent trials
  - Two possible outcomes: drawing a face card (success) and not drawing a face card (failure).

This is a binomial experiment.

Binomial Probability Distribution Function

- Given a binomial experiment,
  - The probability of x successes in n trials where p is the probability of success is

$$P(X = x) = C(n,x)\, p^{x} (1 - p)^{n-x}$$

$$\text{for } x = 0, 1, 2, \ldots, n$$

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$$\text{for } x = 0, 1, 2, \ldots, n$$

Notice that x takes on discrete values, $x = 0, 1, 2, \ldots, n$.

Examples

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Examples

- **Let x be the number of female children in a family having six children. What is the probability that the family has four female children?**
  - **6 trials** **for four females, $x = 4$**
  - **$p = \frac{1}{2}$** **$(1 - p) = \frac{1}{2}$**

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

- Let x be the number of female children in a family having six children. What is the probability that the family has four female children?
  - 6 trials for four females, $x = 4$
  - $p = \frac{1}{2}$ $(1 - p) = \frac{1}{2}$
  - $P(4 \text{ girls}) = C(6,4) \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2$

    $= 15/64$

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

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- A fair coin is tossed seventeen times. What is the probability that the coin lands tail side up five times?
  - 17 trials   for five tails, $x = 5$
  - $p = \frac{1}{2}$ $(1 - p) = \frac{1}{2}$

Examples

- A fair coin is tossed seventeen times. What is the probability that the coin lands tail side up five times?
  - 17 trials   for five tails, $x = 5$
  - $p = \frac{1}{2}$ $(1 - p) = \frac{1}{2}$
  - $P(5 \text{ Tails}) = C(17,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{12}$

$$= 6188 \cdot (1/131072)$$

$$= 1547/32768$$

Examples

- Five cards are drawn with replacement from a fair poker deck. What is the probability that one face card is drawn from the deck?
  - 5 trials   for one face card, x = 1

Examples

- Five cards are drawn with replacement from a fair poker deck. What is the probability that one face card is drawn from the deck?
  - 5 trials for one face card, $x = 1$
  - $p = 3/13$ $(1 - p) = 10/13$

Examples

- Five cards are drawn with replacement from a fair poker deck. What is the probability that one face card is drawn from the deck?
  - 5 trials for one face card, $x = 1$
  - $p = 3/13$ $(1 - p) = 10/13$
  - P(one face card)

$$= C(5,1)\,(3/13)^1\,(10/13)^4$$
$$= 5 \cdot (3/13) \cdot (10^4/13^4)$$

Examples

- Five cards are drawn with replacement from a fair poker deck. What is the probability that one face card is drawn from the deck?
  - 5 trials   for one face card, $x = 1$
  - $p = 3/13$ $(1 - p) = 10/13$
  - P(one face card)

$$= C(5,1)\,(3/13)^1\,(10/13)^4$$
$$= 450000/371293$$

Mean, Variance and Standard Deviation for a Binomial Random Variable

- Given a binomial experiment with n trials and probability of success p,
  - The mean (expected value) is
    $$\mu_X = E(X) = np$$
  - The variance is $\sigma^2_X = np(1 - p)$
  - The standard deviation is $\sigma_X = \sqrt{np(1-p)}$

Examples

- **What is the expected number of female children in a six-child family?**

Examples

- What is the expected number of female children in a six-child family?
  - $n = 6$

Examples

- **What is the expected number of female children in a six-child family?**
  - **n = 6 Since there are six children in the family, there are six independent trials.**
  - **p = ½**

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

- **What is the expected number of female children in a six-child family?**
  - **$n = 6$ Since there are six children in the family, there are six independent trials.**
  - **$p = \frac{1}{2}$**
  - **$\mu_X = E(X) = 6\left(\frac{1}{2}\right)$**
    $= 3$

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

- **What is the expected number of female children in a six-child family?**

- **The expected number of female children is 3.**

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

- **What is the expected number of female children in a six-child family?**

- **The expected number of female children is 3. That is, one can expect to have three female children in a six-child family.**

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

- **What is the expected number of female children in a six-child family?**

- **The expected number of female children is 3. That is, one can expect to have three girls in a six-child family.**

Note: We assume that male and female children are equally likely. If we had evidence to the contrary, we would use that to determine p and 1 – p.

Examples

- What is the standard deviation for the number of female children in a six-child family?

Examples

- What is the standard deviation for the number of female children in a six-child family?
  - $n = 6$
  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$

Examples

- What is the standard deviation for the number of female children in a six-child family?
  - $n = 6$
  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$
  - $\sigma_X = \sqrt{(6(\frac{1}{2})(\frac{1}{2}))}$
    $= \sqrt{(3/2)}$

Examples

- What is the standard deviation for the number of female children in a six-child family?
  - $n = 6$
  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$
  - $\sigma_X = \sqrt{(6(\frac{1}{2})(\frac{1}{2}))}$
    ≈ 1.224744871

Examples

- What is the standard deviation for the number of female children in a six-child family?
  - $n = 6$
  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$
  - $\sigma_X = \sqrt{(6(\frac{1}{2})(\frac{1}{2}))}$
    ≈ 1.224745 girls

Examples

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  - n = 17
  - p = $\frac{1}{2}$

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- What is the expected number of tails in a seventeen-coin toss?
  - $n = 17$
  - $p = \frac{1}{2}$
  - $\mu_X = 17\left(\frac{1}{2}\right)$
    $= 8.5$

Examples

- What is the expected number of tails in a seventeen-coin toss?
  - $n = 17$
  - $p = \frac{1}{2}$
  - $\mu_X = 17\left(\frac{1}{2}\right)$
    $= 8.5$

We can expect 8.5 tails in a 17-coin toss.

Examples

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- What is the standard deviation for the number of tails in a seventeen-coin toss?
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  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$

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- What is the standard deviation for the number of tails in a seventeen-coin toss?
  - $n = 17$
  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$
  - $\sigma_X = \sqrt{(17(\frac{1}{2})(\frac{1}{2}))}$
    $= \sqrt{(4.25)}$

Examples

- **What is the standard deviation for the number of tails in a seventeen-coin toss?**
  - $n = 17$
  - $p = \frac{1}{2}$
  - $1 - p = \frac{1}{2}$
  - $\sigma_X = \sqrt{(17(\frac{1}{2})(\frac{1}{2}))}$
    ≈ 2.061552813 tails

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- If you draw five cards with replacement from a fair poker deck, how many face cards should you expect to receive?
  - n = 5
  - p = 3/13

Examples

- If you draw five cards with replacement from a fair poker deck, how many face cards should you expect to receive?
  - $n = 5$
  - $p = 3/13$
  - $\mu_X = 5(3/13)$
    $= 15/13$
    ≈ 1.153846154

Examples

- If you draw five cards with replacement from a fair poker deck, how many face cards should you expect to receive?
  - n = 5
  - p = 3/13
  - μ_X = 5(3/13)
    = 15/13

We should expect to receive $1\frac{2}{13}$ face cards.

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- Five cards are drawn with replacement from a fair poker deck, what is the standard deviation for the number of face cards?
  - n = 5
  - p = 3/13
  - 1 – p = 10/13

Examples

- Five cards are drawn with replacement from a fair poker deck, what is the standard deviation for the number of face cards?
  - n = 5
  - p = 3/13
  - 1 – p = 10/13
  - $\sigma_X = \sqrt{5\left(\frac{3}{13}\right)\left(\frac{10}{13}\right)}$

Examples

- Five cards are drawn with replacement from a fair poker deck, what is the standard deviation for the number of face cards?
  - n = 5
  - p = 3/13
  - 1 – p = 10/13
  - $\sigma_X = \sqrt{\frac{150}{169}}$

Examples

- Five cards are drawn with replacement from a fair poker deck, what is the standard deviation for the number of face cards?
  - $n = 5$
  - $p = 3/13$
  - $1 - p = 10/13$
  - $\sigma_X \approx 0.9421114395$ face cards