Confidence Intervals

Inferential Statistics

- The process of using information obtained using a sample and generalizing and using this information to draw conclusions about or to make generalizations about a population

Recall

- Parameters – calculations made from population data
  - For example, the population mean, μ, and the population standard deviation, σ
- Statistics – calculations made from sample data
  - For example the sample mean, $\overline{X}$, and the sample standard deviation, s

Estimation

- Using sample data to estimate a value for an unknown parameter such as the population mean, μ, or the population standard deviation, σ

Central Limit Theorem

- Given a random variable X with population mean μ and (population) standard deviation σ, for a random sample of size n taken from this population, the sampling distribution of $\overline{X}$ becomes approximately normal as the sample size n increases. The mean of the distribution is $\mu_{\bar{x}} = \mu$ and the standard deviation is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

Point Estimate

- A point estimate for a *parameter*
  - The value of a *statistic* that estimates the value of the *parameter*
- Example: sample mean, $\overline{X}$, is a point estimate for the population mean, μ

Other Point Estimates for the Population Mean

- In addition to using the sample mean, $\overline{X}$, as a point estimate for the population mean, μ, we could use
  - Sample median
  - Mode

Other Point Estimates for the Population Mean

- In addition to using the sample mean, $\overline{X}$, as a point estimate for the population mean, μ, we could use
  - Sample median
  - Mode
- Any measure of center

Other Point Estimates for the Population Mean

- In addition to using the sample mean, $\overline{X}$, as a point estimate for the population mean, μ, we could use
  - Sample median
  - Mode
- Q: Which point estimate should be used to estimate the population mean?

Use Sample Mean as Point Estimate for Population Mean

- Unbiased estimator: *A statistic is an* ***unbiased estimator*** if its expected value is the same as the value of the parameter.

Use Sample Mean as Point Estimate for Population Mean

- Unbiased estimator: *A* statistic is an *unbiased estimator* if its expected value is the same as the value of the parameter. *Recall* – a statistic is unbiased if it does not systematically *overestimate* or *underestimate* the value of the parameter that it estimates

Use Sample Mean as Point Estimate for Population Mean

- Unbiased estimator: A statistic is an *unbiased estimator* if its expected value is the same as the value of the parameter. ***Recall*** **– by Central Limit Theorem, if we took many samples of size n and calculated the mean for each sample then the mean of the sample means would be μ**

Use Sample Mean as Point Estimate for Population Mean

- **Consistent:** the sample mean provides consistent estimates of the population mean since *as the sample size increases*, the value of the sample mean gets *closer* to the value of the population mean

Use Sample Mean as Point Estimate for Population Mean

- Efficiency: the majority of sample means will be "close" to the value of the population mean

Best Point Estimate

- The sample mean, $\overline{x}$, is the *best point estimate* for the population mean, μ

Caution

- In using point estimates, we *never* know
  - If the statistic is *correct*
- We can determine if the *procedure* used to produce the statistic is
  - *Unbiased*
  - *Consistent*
  - *Efficient*

Confidence Interval

- **A confidence interval estimate of a parameter consists of an interval of numbers together with a measure of the likelihood that the interval contains the unknown parameter**

Confidence Interval

- A confidence interval estimate of a parameter consists of an interval of numbers together with a measure of the likelihood, the Level of Confidence, that the interval contains the unknown parameter

Confidence Interval

- A confidence interval estimate of a parameter consists of *a pair of numbers* with an associated probability, the Level of Confidence, that the unknown parameter *will lie between the two numbers*

Level of Confidence

- The level of confidence in a confidence interval is the percentage of intervals that will contain μ if a large number of repeated samples are taken.

Level of Confidence

- The level of confidence in a confidence interval is the percentage of intervals that will contain μ if a large number of repeated samples are taken.
- Associated formula: $(1 - \alpha).100\%$

Values of α in $(1 - \alpha) \cdot 100\%$

Level of Confidence $(1 - \alpha) \cdot 100\%$	Value of α
90%	0.1
95%	0.05
96%	0.04
99%	0.01

Values of α in $(1 - \alpha)\cdot 100\%$

Level of Confidence $(1 - \alpha)\cdot 100\%$	Value of α	Area in each "Tail", $\alpha/2$
90%	0.1	0.05
95%	0.05	0.025
96%	0.04	0.02
99%	0.01	0.005

Values of α in (1 - α)·100%

Level of Confidence (1 – α)•100%	Value of α	Area in each "Tail", α/2	Critical Value, $Z_{\alpha/2}$
90%	0.1	0.05	1.645
95%	0.05	0.025	1.96
96%	0.04	0.02	2.05*
99%	0.01	0.005	2.575

*Note: Value from table of Standard Normal Probabilities.

Values of α in (1 − α)·100%

Level of Confidence $(1-\alpha)\cdot 100\%$	Value of α	Area in each "Tail", $\alpha/2$	Critical Value, $Z_{\alpha/2}$
90%	0.1	0.05	1.645
95%	0.05	0.025	1.96
96%	0.04	0.02	2.053748911
99%	0.01	0.005	2.575

***Note: Value obtained using TI-84 Plus Silver Edition.**

Values of α in $(1 - \alpha)\cdot 100\%$

Level of Confidence $(1 - \alpha)\cdot 100\%$	Value of α	Area in each "Tail", $\alpha/2$	Critical Value, $Z_{\alpha/2}$
90%	0.1	0.05	1.645
95%	0.05	0.025	1.96
96%	0.04	0.02	2.054
99%	0.01	0.005	2.575

***Note: Approximate value expressed to three decimal places**

Interpretation of Confidence Interval

- A $(1 - \alpha) \cdot 100\%$ confidence interval tells us that *if* we were to obtain many simple random samples of size n from a population for which the mean μ is unknown *then* approximately $(1 - \alpha) \cdot 100\%$ of the intervals would contain the value of the population mean μ.

Central Limit Theorem

- Given a random variable X with population mean μ and (population) standard deviation σ, for a random sample of size n taken from this population, the sampling distribution of $\overline{X}$ becomes approximately normal as the sample size n increases. The mean of the distribution is $\mu_{\overline{x}} = \mu$ and the standard deviation is $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$.

By the Central Limit Theorem

- We know that
  - The distribution for the sample means is approximately normal
  - The standard deviation for the sample means is $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$ for known population standard deviation σ

Constructing a $(1 - \alpha)\cdot 100\%$ Confidence Interval about μ with Known Population Standard Deviation σ

- Suppose a simple random sample size n is taken from a population with unknown mean μ and known standard deviation σ.
- A $(1 - \alpha)\cdot 100\%$ confidence interval for μ is determined by
  - Lower bound: $\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
  - Upper bound: $\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

  for $z_{\alpha/2}$ the critical z-value

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

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- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. *Assuming* the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- Note: It is not realistic to know the population standard deviation especially when we are trying to determine the confidence interval for the population mean.

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- We know the values for the sample size, $n = 1120$, the sample mean, $\bar{x} = 8.17$, and the population standard deviation, $\sigma = 1.2$.

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- For a 95% confidence interval, $\alpha = 0.05$ and $\alpha/2 = 0.025$.

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- For a 95% confidence interval, $\alpha = 0.05$
  and $\alpha/2 = 0.025$.
- Then, $z_{\alpha/2} = 1.96$ is the critical value.

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- Using these values ($n = 1120$, $\bar{x} = 8.17$, $\sigma = 1.2$, and $z_{\alpha/2} = 1.96$), we calculate the lower bound and the upper bound for the confidence interval.

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **Lower bound:** $\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 8.17 - 1.96\frac{1.2}{\sqrt{1120}}$

$$\approx 8.099720558$$

$$\approx 8.10$$

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- Upper bound: $\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 8.17 + 1.96\frac{1.2}{\sqrt{1120}}$

$$\approx 8.240279442$$
$$\approx 8.24$$

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **Confidence Interval:**
  - **$8.10 \leq \mu \leq 8.24$ (Inequality Notation)**
  - **$\mu \in [8.10, 8.24]$ (Interval Notation)**

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **We are 95% confident that the population mean, μ, is such that $8.10 \le \mu \le 8.24$.**

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **We are 95% confident that the population mean, μ, is in the interval [8.10, 8.24].**

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **We are 95% confident that the population mean, μ, is between 8.10 hours and 8.24 hours, inclusive.**

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **We are 95% confident that the population mean, μ, is between 8.10 hours and 8.24 hours, inclusive.**

This would be a meaningful statement for the confidence interval.

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **We are 95% confident that the population mean, μ, is between 8.10 hours and 8.24 hours, inclusive.**

- ***What does this mean?***

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- Our statement tells us that if we were to select many different samples of size 1120 and construct the corresponding confidence intervals, 95% of these confidence intervals would actually contain the value of the population mean.

- Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.

- Note: The level of confidence, in this case 95%, refers to the success rate of the process being used to estimate the mean.

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **CAUTION: The level of confidence, in this case 95%, is <u>NOT</u> a probability!!! It would be <u>INCORRECT</u> to say that 95% of sample means would be between 8.10 hours and 8.24 hours, inclusive.**

- **Time in Bed: How much do Americans sleep each night? Based on a random sample of 1120 Americans 15 years of age or older, the mean amount of sleep per night is 8.17 hours according to the American Time Use Survey conducted by the Bureau of Labor Statistics. Assuming the population standard deviation for the amount of sleep per night is 1.2 hours, construct a 95% confidence interval for the mean amount of sleep per night for Americans 15 years of age or older.**

- **CAUTION: The level of confidence, in this case 95%, is <u>NOT</u> a probability!!! It would be <u>WRONG</u> to say that the probability that the population mean is between 8.10 hours and 8.24 hours, inclusive, is 0.95.**

Margin for Error

- **The margin for error in a confidence interval for which σ is known is determined by**

$$E = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

for sample size n.

Margin for Error

- The margin for error in a confidence interval for which σ is known is determined by

$$E = z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

for sample size n.

- Note: The distribution must be normal or the sample size must be at least 30.

Margin for Error

- If we know the margin for error,

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

that we are willing to tolerate then we can determine the sample size necessary to achieve the desired margin for error.

Margin for Error

- **If we know the margin for error,**

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

that we are willing to tolerate then we can determine the sample size necessary to achieve the desired margin for error.

- HOW???

Margin for Error

- If we know the margin for error that we are willing to tolerate then we can determine the sample size necessary to achieve the desired margin for error.

- Solve $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ for n.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$E\sqrt{n} = z_{\alpha/2} \cdot \sigma$$

Multiply both sides by the square root of n and divide both sides by E.

$$\sqrt{n} = \frac{z_{\alpha/2} \cdot \sigma}{E}$$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

Square both sides and, to simplify the expression, square each value on the right-hand side, if desired. Either form can be used.

$$n = \frac{\left(z_{\alpha/2} \right)^2 \cdot \sigma^2}{E^2}$$

Determining the Sample Size n

- The sample size n required to estimate the population mean μ with a level of confidence of $(1 - \alpha)\cdot 100\%$ with a specified margin of error E is determined by

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

where n is rounded up to the nearest whole number.

- **In “real life” will we know the population standard deviation???**

- In “real life” will we know the population standard deviation???

Most likely not!

- **How Much TV Do Teenagers Watch? A Gallup poll conducted January 17 – February 6, 2005, asked 1028 teenagers 13 years of age to 17 years of age , "Typically, how many hours per week do you spend watching TV?" The teenagers who participated in the survey watched, on average, 13.0 hours of television per week with a standard deviation of 2.3 hours per week. Construct a 95% confidence interval for the average number of hours that teenagers watch TV each week.**

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- We would like to construct a 95% confidence interval.

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- For a 95% confidence interval, $\alpha = 0.05$
  and $\alpha/2 = 0.025$

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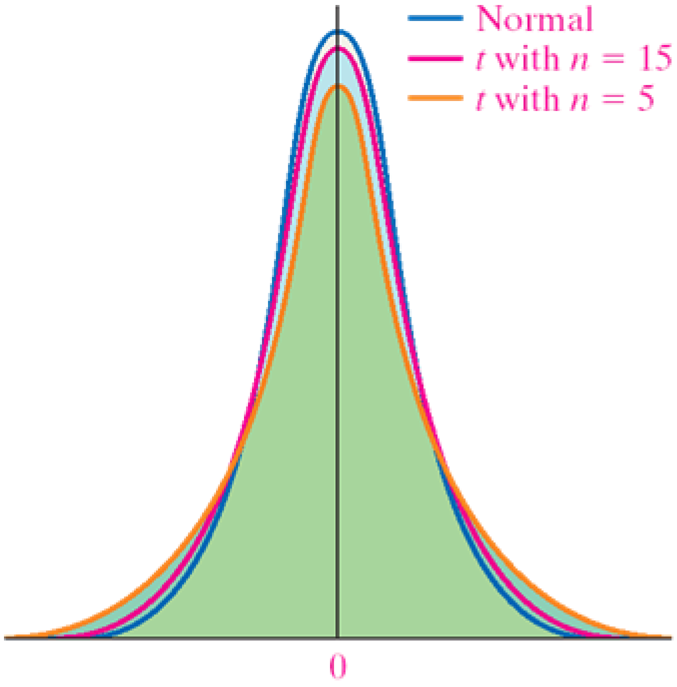
- We know the values for the sample size, $n = 1028$, the sample mean, $\bar{x} = 13.0$, and the sample standard deviation, $s = 2.3$.

- **How Much TV Do Teenagers Watch? A Gallup poll conducted January 17 – February 6, 2005, asked 1028 teenagers 13 years of age to 17 years of age , "Typically, how many hours per week do you spend watching TV?" The teenagers who participated in the survey watched, on average, 13.0 hours of television per week with a standard deviation of 2.3 hours per week. Construct a 95% confidence interval for the average number of hours that teenagers watch TV each week.**

- **We use the t-distribution to determine the critical value for the calculation of the lower bound and the upper bound.**

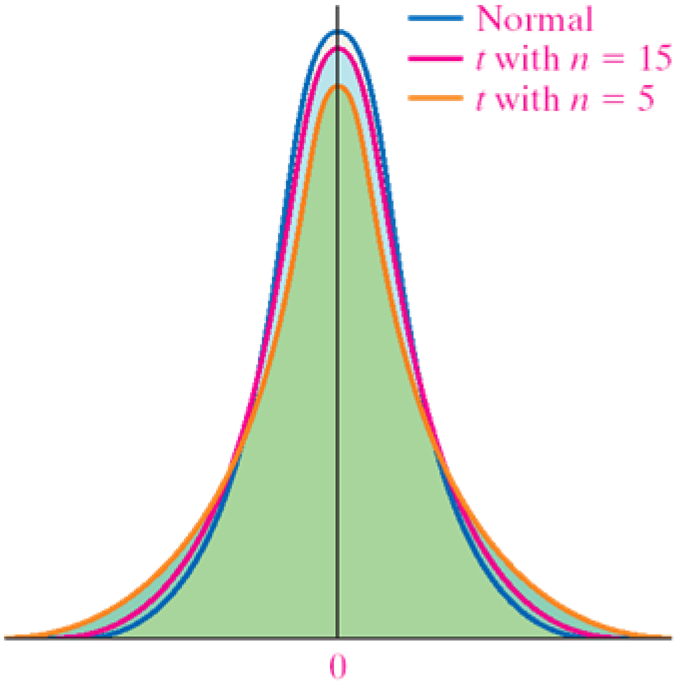
Properties of t-Distribution

- The t-Distribution changes based on the number of degrees of freedom



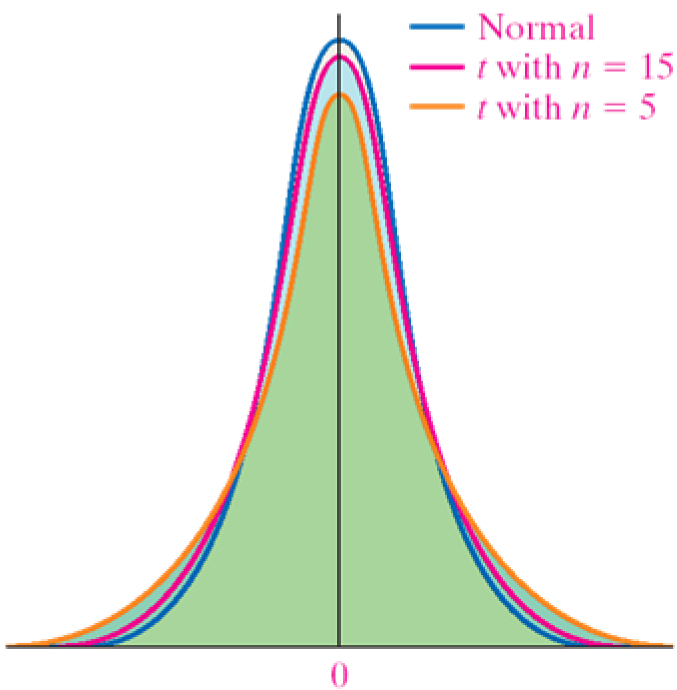
Properties of t-Distribution

- The t-Distribution is centered at 0 and is symmetric about 0.



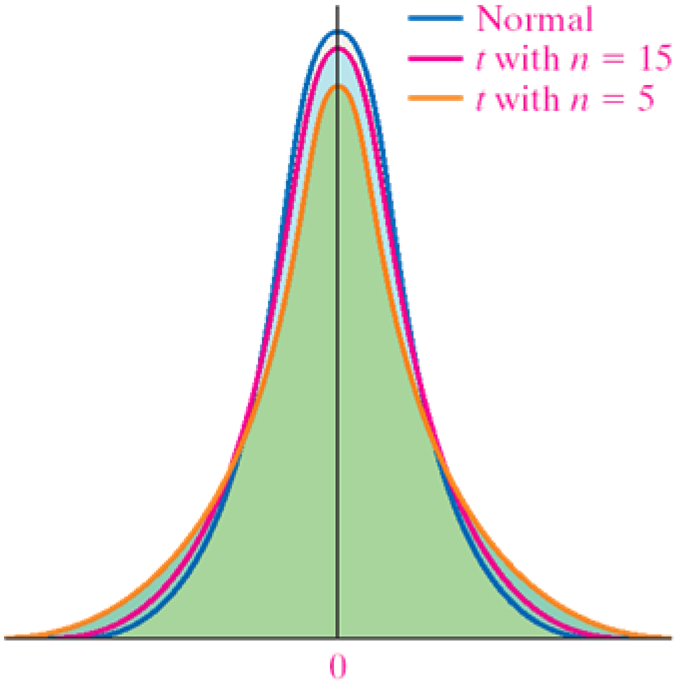
Properties of t-Distribution

- The area under the curve is 1.



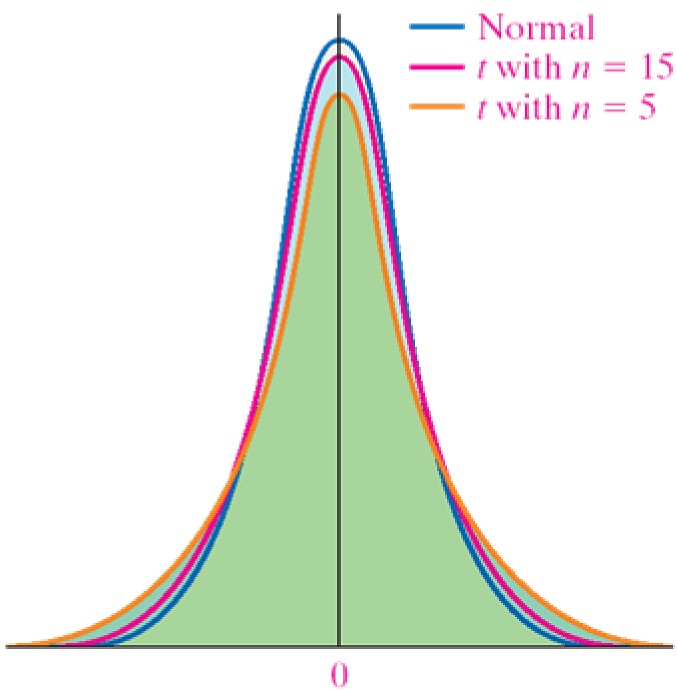
Properties of t-Distribution

- The area under the curve is to the left of 0 is ½.



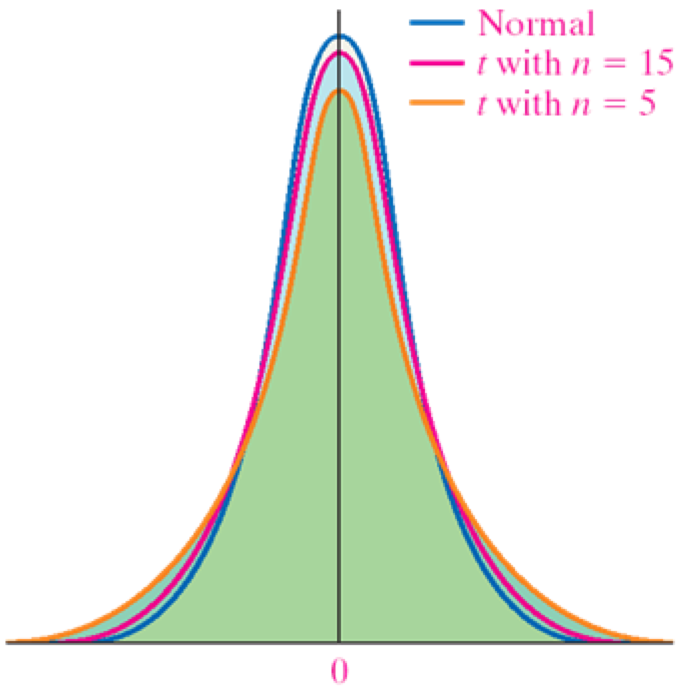
Properties of t-Distribution

- The area under the curve is to the right of 0 is ½.



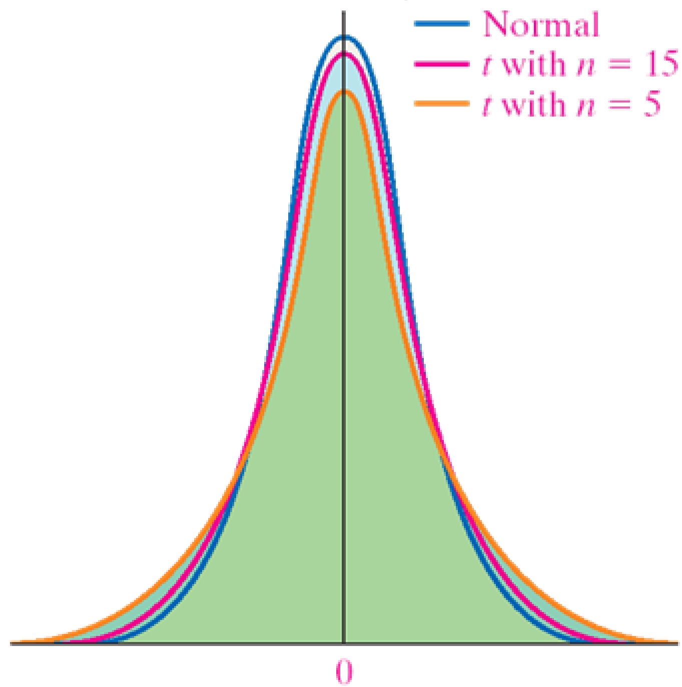
Properties of t-Distribution

- The curve never touches the horizontal axis.



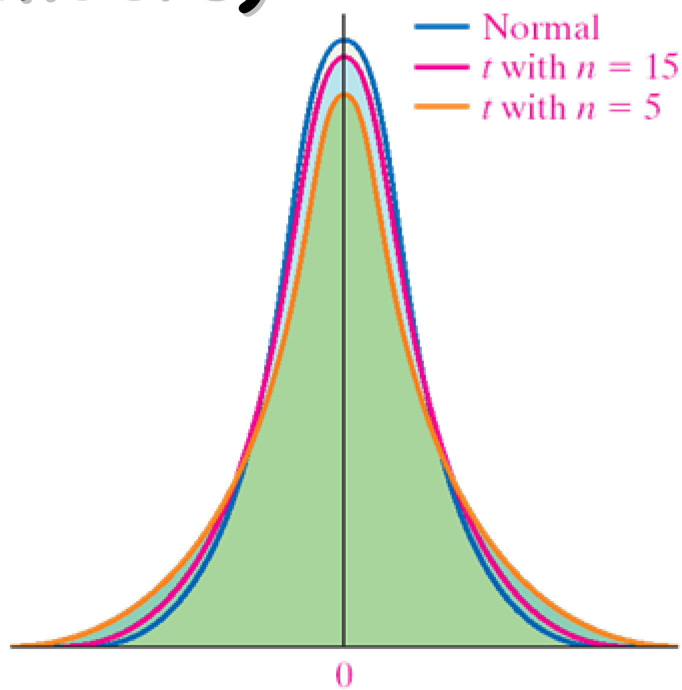
Properties of t-Distribution

- The area in the "tails" of the t-Distribution is a little *greater* than the area in the tails of the standard normal distribution.



Properties of t-Distribution

- As the sample size n increases, the density curve of t gets closer to the standard normal density curve. (Law of Large Numbers)



Constructing a $(1 - \alpha)\cdot 100\%$ Confidence Interval about μ and σ unknown

- Suppose a simple random sample size n is taken from a population with unknown mean μ and an unknown standard deviation σ.
- A $(1 - \alpha)\cdot 100\%$ confidence interval for μ is determined by
  - Lower bound: $\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}}$
  - Upper bound: $\bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$

  for $t_{\alpha/2}$ computed with n-1 degrees of freedom

- How Much TV Do Teenagers Watch? A Gallup poll conducted January 17 – February 6, 2005, asked 1028 teenagers 13 years of age to 17 years of age , "Typically, how many hours per week do you spend watching TV?" The teenagers who participated in the survey watched, on average, 13.0 hours of television per week with a standard deviation of 2.3 hours per week. Construct a 95% confidence interval for the average number of hours that teenagers watch TV each week.

- Since n = 1028, there are 1027 degrees of freedom. Since our table does not list 1027 degrees of freedom, we will use the next closest, 1000 degrees of freedom.

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- The critical value corresponding to $\alpha = 0.025$ and 1000 degrees of freedom is $t_{\alpha/2} = 1.962$.

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- **We use our values n = 1028, $\bar{x}$ = 13.0, s = 2.3, and $t_{\alpha/2}$ = 1.962 to determine the lower bound and the upper bound for the confidence interval.**

- **How Much TV Do Teenagers Watch? A Gallup poll conducted January 17 – February 6, 2005, asked 1028 teenagers 13 years of age to 17 years of age , "Typically, how many hours per week do you spend watching TV?" The teenagers who participated in the survey watched, on average, 13.0 hours of television per week with a standard deviation of 2.3 hours per week. Construct a 95% confidence interval for the average number of hours that teenagers watch TV each week.**

- **Lower bound:** $\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} = 13.0 - 1.962\frac{2.3}{\sqrt{1028}}$

$$\approx 12.85925587$$
$$\approx 12.9$$

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- **Upper bound:** $\bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}} = 13.0 + 1.962\frac{2.3}{\sqrt{1028}}$

$$\approx 13.14074413$$
$$\approx 13.1$$

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- Confidence Interval:
  - $12.9 \leq \mu \leq 13.1$ (Inequality Notation)
  - $\mu \in [12.9, 13.1]$ (Interval Notation)

- How Much TV Do Teenagers Watch? A Gallup poll conducted January 17 – February 6, 2005, asked 1028 teenagers 13 years of age to 17 years of age , “Typically, how many hours per week do you spend watching TV?” The teenagers who participated in the survey watched, on average, 13.0 hours of television per week with a standard deviation of 2.3 hours per week. Construct a 95% confidence interval for the average number of hours that teenagers watch TV each week.

- We are 95% confident that the population mean, μ, is such that $12.9 \leq \mu \leq 13.1$.

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- **We are 95% confident that the population mean is between 12.9 hours per week and 13.1 hours per week, inclusive.**

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- We are 95% confident that the population mean is between 12.9 hours per week and 13.1 hours per week, inclusive.
- *What does this mean?*

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- **The statement tells us that if we were to slect many different samples of size 1028 and construct the corresponding confidence intervals, 95% of these confidence intervals would actually contain the population mean.**

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- **Again, the 95% is the success rate of the process being used to estimate the population mean.**

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- Note: The 95% is NOT a probability.

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- **CAUTION: Saying that there is a 95% chance that the population mean is in the confidence interval is <u>INCORRECT</u>!!!**

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- **CAUTION: It is <u>WRONG</u> to say that 95% of sample means are in the confidence interval.**

Choosing between the Normal Distribution and the t-Distribution for Confidence Intervals

Use Normal Distribution (z)	σ known *and* population normally distributed OR σ known *and* n ≥ 30
Use t-Distribution (t)	σ not known *and* population normally distributed OR σ not known *and* n ≥ 30

Round-off Rule for Confidence Intervals used to Estimate μ

- When using original sample data to construct a confidence interval, round the limits of the confidence interval to one more decimal place than the original data.
- When using given values of $\bar{x}$, n, and s with the sample data unknown, round the limits of the confidence interval to the same number of decimal places as the sample mean.