Interpreting the Slope and the y-Intercept

- What is the meaning of the slope?
- What is the meaning of the y-intercept?

What must one consider when collecting data?

- What is being measured?
- Is there one variable or more than one variable?
  - What is (are) the unit(s) for the variable(s)?
- For what or how will the data be used?
- What does the data mean?

What are some considerations for data collection?

- Understand what is being measured?
- Determine how many variables must be defined?
  - Determine the unit(s) for the variable(s).
- Determine how the data is to be collected. Choose the measurement device(s) carefully.
- Determine the amount of data to be collected.

How could we determine the thickness of the hard cover of a textbook and the thick ness of a page in a textbook using an ordinary ruler?

- Measurement device: a transparent six-inch ruler with fine measurements to 1/32 inch as well as millimeters
- Measure the thickness of the front cover with various amounts of pages from the text: create linear function and use function to determine desired information

How could we determine the thickness of the hard cover of a textbook and the thick ness of a page in a textbook using an ordinary ruler?

- Problems –
  - How one holds the ruler
    - Edge from which one measures the thickness
      - Best to make consistent measurements

How could we determine the thickness of the hard cover of a textbook and the thick ness of a page in a textbook using an ordinary ruler?

- Problems –
  - How tightly one holds/pinches the pages with the front cover
    - Human error?
  - Most hard cover books have an end page
    - Tear out or keep in?

How could we determine the thickness of the hard cover of a textbook and the thick ness of a page in a textbook using an ordinary ruler?

- Question – Why "sheets of paper" rather than "pages"?
- Answer – A sheet of paper in a book has two pages, one on each side. A sheet of paper has thickness that can be determined.

How could we determine the thickness of the hard cover of a textbook and the thick ness of a page in a textbook using an ordinary ruler?

- Considerations – One must be careful to count the pieces of paper rather than relying on page numbers.
  - Some pages in a book may be numbered with Roman numerals or with special page numbering systems.

The data on the next slide is presented for your consideration. The data was carefully collected and all measurements were made with a transparent ruler with the described attributes.

- Note: Measurements of the thickness were made in inches as well as in centimeters.

Pinching Pages Activity –

Measurement made using *Statistics in Action – Understanding a World of Data,* Ann E. Watkins, Richard L. Scheaffer, George W. Cobb, Key Curriculum Press, 2004 (ISBN: 1-931914-27-3)

# sheets of paper	Thickness, in	Thickness, cm
50	0.25	0.7
100	0.375	1.0
150	0.5	1.3
200	0.625	1.6
250	0.75	1.9

What does this data mean?

- The first column is the number of sheets of paper used in each measurement.
- The thicknesses measured, in inches in the second column and in centimeters in the third column, are the thickness of the front cover, one end page, and the number of sheets of paper specified in the first column.

What is x and what is y?

- Since the thickness measured depends on the number of sheets of paper involved in the measurement, the number of sheets of paper is the independent variable, x, and the thickness of the front cover, the end page, and the specified number of sheets of paper is the dependent variable, y.

Define y in terms of x - general

- Y is the thickness of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*

Define y in terms of x – for first and second columns

- Y is the thickness, in inches, of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*

Define y in terms of x – for first and third columns

- Y is the thickness, in centimeters, of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*

Define y in terms of x

- Question: Can you see the differences among the following three statements?
  - Y is the thickness of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*
  - Y is the thickness, in inches, of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*
  - Y is the thickness, in centimeters, of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*

Define y in terms of x

- Question: Can you see the differences among the following three statements?
  - Y is the thickness of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*
  - Y is the thickness, in inches, of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*
  - Y is the thickness, in centimeters, of the front cover, one end page and x sheets of paper of the textbook *Statistics in Action – Understanding a World of Data.*

Pinching Pages Activity –

Measurement made using *Statistics in Action – Understanding a World of Data,* Ann E. Watkins, Richard L. Scheaffer, George W. Cobb, Key Curriculum Press, 2004 (ISBN: 1-931914-27-3)

- Using the data below, determine the least squares line that can be used to determine the thickness of one cover, the end page, and x sheets of paper in "Statistics in Action – Understanding a World of Data"
  - measured in centimeters AND
  - measured in inches

x	y (in)	y (cm)
50	0.25	0.7
100	0.375	1
150	0.5	1.3
200	0.625	1.6
250	0.75	1.9

Given the data

- We know how to determine **m**.
- We know how to determine **b**.

...

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# sheets of paper	Thickness, in
50	0.25
100	0.375
150	0.5
200	0.625
250	0.75

- m = 0.0025
- b = 0.125
- r = 1

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- m = 0.0025
- b = 0.125
- r = 1
- *What does this mean???*

Pinching Pages Activity –

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# sheets of paper	Thickness, cm
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200	1.6
250	1.9

- m = 0.006
- b = 0.4
- r = 1

Pinching Pages Activity –

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- m = 0.006
- b = 0.4
- r = 1
- *What does this mean???*

What are the units on the slope?

Since the slope is the ratio of the vertical change, the rise, to the horizontal change, the run,

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$

the units on the slope are

$$\frac{\text{units on the rise}}{\text{units on the run}} = \frac{\text{y-units}}{\text{x-units}}$$

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- m = 0.0025
- What are the units on the slope?

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200	0.625
250	0.75

- m = 0.0025
- What are the units on the slope?
  - inches per sheet

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# sheets of paper	Thickness, cm
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100	1.0
150	1.3
200	1.6
250	1.9

- m = 0.006
- What are the units on the slope?
  - centimeters per sheet

What are the units on the y-coordinate of the y-intercept?

- Since the y-coordinate of the y-intercept is a y-value, the units on the y-coordinate of the y-intercept are y-units.

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- b = 0.125
- What are the units on b?

Pinching Pages Activity –

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- b = 0.125
- What are the units on b?
  - inches

Pinching Pages Activity –

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- b = 0.4
- What are the units on b?

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100	1.0
150	1.3
200	1.6
250	1.9

- b = 0.4
- What are the units on b?
  - centimeters

Interpret the slope

- Keep in mind that the slope is the quotient of the change in y and the change in x
  - This is the average rate of change.
    - Rate of change on an interval

Interpret the slope

- Keep in mind that the slope is the quotient of the change in y and the change in x
  - This is the average rate of change.
    - Rate of change on an interval
- How can we make the interpretation as simple as possible?

Interpret the slope

- Keep in mind that the slope is the quotient of the change in y and the change in x
  - This is the average rate of change.
    - Rate of change on an interval
- How can we make the interpretation as simple as possible? Choose a simple x-interval (change in x)

Interpret the slope

- Keep in mind that the slope is the quotient of the change in y and the change in x
  - This is the average rate of change.
    - Rate of change on an interval
- What is the simplest change in x?

Interpret the slope

- Keep in mind that the slope is the quotient of the change in y and the change in x
  - This is the average rate of change.
    - Rate of change on an interval
- What is the simplest change in x?
  A one-unit change in x

Let's interpret the slope

- Thickness measured in inches
  - Units: inches per sheet
  - m = 0.0025 expressed as a fraction with a denominator of 1: 0.0025/1 where
    - 1 is the change in the number of sheets of paper
    - A positive change is an increase
    - One *additional* page

Let's interpret the slope

- Thickness measured in inches
  - Units: inches per sheet
  - m = 0.0025 expressed as a fraction with a denominator of 1: 0.0025/1 where
    - 0.0025 is the change in the thickness of the front cover, one end page and the corresponding number of sheets of paper
    - A positive change is an increase
    - The thickness of the front cover, one end page, and the corresponding number of sheets of paper increases by 0.0025 inches

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and (a number of or several) sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* increases by 0.0025 inch for each additional page.

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and (a number of or several) sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* increases by 0.0025 inch for each additional page.
- This is awkward. Can this be simplified?

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and (a number of or several) sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* increases by 0.0025 inch for each additional page.
- This is awkward. Can this be simplified? **Yes!**

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and sheets of paper of the textbook Statistics in Action – Understanding a World of Data increases by 0.0025 inches for each additional page.
- By what is the measurement changed?

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and sheets of paper of the textbook Statistics in Action – Understanding a World of Data increases by 0.0025 inch for each additional page.
- By what is the measurement changed?
  - By the one page
  - By a thickness of 0.0025 inch

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and sheets of paper of the textbook Statistics in Action – Understanding a World of Data increases by 0.0025 inch for each additional page.
- By what is the measurement changed?
  - By the one page
  - By a thickness of 0.0025 inch
- If we add one sheet of paper and change the thickness by 0.0025 inch, is this the thickness of the sheet of paper?

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and sheets of paper of the textbook Statistics in Action – Understanding a World of Data increases by 0.0025 inch for each additional page.
- By what is the measurement changed?
  - By the one page
  - By a thickness of 0.0025 inch
- If we add one sheet of paper and change the thickness by 0.0025 inch, is this the thickness of the sheet of paper? YES!

Let's interpret the slope

- Combine these ideas to interpret the slope of m = 0.0025 and put it in context
  - The thickness of the front cover, one end page and sheets of paper of the textbook Statistics in Action – Understanding a World of Data increases by 0.0025 inch for each additional page.
- By what is the measurement changed?
  - By the one page
  - By a thickness of 0.0025 inch
- Simple interpretation?

Let's interpret the slope

- By what is the measurement changed?
  - By the one sheet of paper
  - By a thickness of 0.0025 inch
- Simple interpretation?
  - The thickness of one sheet of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.0025 inch.

Let's interpret the slope

- By what is the measurement changed?
  - By the one sheet of paper
  - By a thickness of 0.0025 inch
- Simple interpretation?
  - The thickness of one sheet of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.0025 inch.
- Do all slopes have two interpretations?

Let's interpret the slope

- By what is the measurement changed?
  - By the one sheet of paper
  - By a thickness of 0.0025 inch
- Simple interpretation?
  - The thickness of one sheet of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.0025 inch.
- Do all slopes have two interpretations? NO

Let's interpret the y-intercept for the least squares line.

Let's interpret the y-intercept for the least squares line.

- Why the y-intercept of the least squares line instead of the y-coordinate of the y-intercept of the least squares line?

Let's interpret the y-intercept for the least squares line.

- Why the y-intercept of the least squares line instead of the y-coordinate of the y-intercept of the least squares line?
  - The y-value depends on the x-value.
  - We cannot interpret a y-value without its corresponding x-value.

Let's interpret the y-intercept for the least squares line.

- So, what is the corresponding x-value?

Let's interpret the y-intercept for the least squares line.

- So, what is the corresponding x-value?
  - Two ways to remember this:
    - Since on the y-axis, x is equal to zero, the coordinates of the y-intercept are (0, b).
    - Given the equation, $y = mx + b$, what x do we need so that y will be equal to b? We need $x = 0$. So, again, the coordinates of the y-intercept are (0, b).
  - That is, when $x = 0$ then $y = b$.

Let's interpret the y-intercept for the least squares line.

- Does it matter if we say "when x = 0 then y = b"?

Let's interpret the y-intercept for the least squares line.

- Does it matter if we say "when x = 0 then y = b"? YES!
  - The value of y depends on x.
  - x is the independent variable, the free variable, the explanatory variable. The x-value is free to change.
  - y is the dependent variable, the response variable. The y-value depends on the x-value. The y-value responds to changes in the x-value.

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - What is the y-intercept?

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - What is the y-intercept? (0, 0.125)

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - What is the y-intercept? (0, 0.125)
  - The interpretation (meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover, one end page and 0 sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - The interpretation (meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover, one end page and 0 sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - The interpretation (meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover, one end page and 0 sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.
  - Can this be simplified?

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - The interpretation (meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover, one end page and 0 sheets of paper of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.
  - Can this be simplified? YES!

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - The simple interpretation (simple meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover and one end page of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - The simple interpretation (simple meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover and one end page of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.
  - Do all y-intercepts have two interpretations?

Let's interpret the y-intercept for the least squares line.

- Interpret the y-intercept that corresponds to knowing that b = 0.125 and put it in context.
  - The simple interpretation (simple meaning) of the y-intercept (0, 0.125)
    - The thickness of the front cover and one end page of the textbook *Statistics in Action – Understanding a World of Data* is 0.125 inch.
  - Do all y-intercepts have two interpretations? NO

Use the Least Squares Line

- Using this data, can one determine the thickness of one (front) cover of the text *Statistics in Action – Understanding a World of Data*?

Use the Least Squares Line

- Using this data, can one determine the thickness of one (front) cover of the text *Statistics in Action – Understanding a World of Data*?
- NO

Use the Least Squares Line

- Using this data, can one determine the thickness of one (front) cover of the text *Statistics in Action – Understanding a World of Data*? NO
  - Why?

Use the Least Squares Line

- Using this data, can one determine the thickness of one (front) cover of the text *Statistics in Action – Understanding a World of Data*? NO
  - Why?
    - The thickness measurements include the thickness of the *front end page*, and this cannot removed from the data.

Use the Least Squares Line

- Using this data, can one determine the thickness of one (front) cover of the text *Statistics in Action – Understanding a World of Data*? NO
  - Why include the front end page?

Use the Least Squares Line

- Using this data, can one determine the thickness of one (front) cover of the text *Statistics in Action – Understanding a World of Data*? NO
  - Why include the front end page?
    - So that I could bring up that point that one cannot remove this unknown quantity (thickness of the front end page) from our data.

Use the Least Squares Line

- What have we determined from this least squares line?

Use the Least Squares Line

- What have we determined from this least squares line?
  - The thickness of one sheet of paper of the textbook *Statistics in Action – Understanding a World of Data.*
  - The thickness of the front cover and the front end page of the textbook *Statistics in Action – Understanding a World of Data.*

Pinching Pages Activity –

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- r = 1
- *What does r = 1 mean???*

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250	0.75

- r = 1
- *What does r = 1 mean???*
  - The data is linear.
  - The line fits the data.

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