What is in this Handout: This handout contains information related to MATH 180 Precalculus. Its purpose is to help you make an informed decision about which course in the Algebra- Precalculus-Calculus sequence is right for you at Framingham State. Below you will find the catalog description for these courses, and a general overview of MATH 180 Precalculus. The next page provides a set of competency exercises that could typically appear in a MATH 180 Precalculus course. Students who feel confident in their ability to complete *almost all* of these exercises may wish to learn more about the subsequent course, MATH 219 Calculus I, to best determine their appropriate starting point in the Algebra-Precalculus-Calculus sequence. Students with no exposure to calculus in high school are encouraged to enroll in either MATH 123 College Algebra or MATH 180 Precalculus.

The Algebra-Precalculus-Calculus Sequence:

MATH 123 College Algebra: This course is an exploration of numerical, graphical and symbolic approaches to algebraic concepts with emphasis on real-world applications, modeling, and problem-solving skills. Topics include polynomials, rational expressions, equations and inequalities, systems of linear equations, matrices, and the connection between functions and their graphs.

MATH 180 Precalculus: This course is a thorough preparation in the skills and topics needed to study calculus. After a review of polynomial and rational functions and their graphs, topics include inverse functions, exponential and logarithmic functions, and trigonometric functions.

MATH 219 Calculus I: This course is a study of functions, limits, continuity, the derivative, rules of differentiation of algebraic, trigonometric, exponential and logarithmic functions, applications of differentiation, definite and indefinite integrals, and the Fundamental Theorem of Calculus.

General Overview: MATH 180 Precalculus is a continuation of MATH 123 College Algebra and typically begins with a quick review of quadratic and polynomial functions. Rational functions are introduced and developed completely to complete the review content. Rational and radical functions provide an introduction to inverse functions which are a central theme in the remainder of the course. Exponential and logarithmic functions are developed with emphasis on their applications to the natural sciences and finance, as well as their mathematical utility. Properties of exponential and logarithmic functions and their graphs are studied with focus on the inverse relationship between the two classes of functions.

The final class of functions in MATH 180 Precalculus is the trigonometric functions, which are developed from an analytic perspective and studied as functions of a real variable. This complete development includes: evaluation, properties and behaviors, analysis of graphs - simple and transformations, geometric interpretations, solving analytic equations, inverse trigonometric functions, and construction and establishment of identities. Within each of the three classes of functions listed above, the concepts of global and local behavior are heavily emphasized as part of a more complete analysis of a function.

Precalculus is designed to provide students with the knowledge of functions required for Calculus; as these functions are also of great importance to the natural sciences, students may move directly into a science course after completion of MATH 180 Precalculus. Therefore, a primary focus throughout much of the course is on developing mathematical justification for conclusions; processes and procedures are comprehensive and require support at each stage. Adherence to rigorous computation is required, and more formal notation is introduced with expected implementation.

Rational Functions

Let,

$$f(x) = \frac{2x^2 + 5x - 3}{(x^2 - 4)(x + 5)}.$$

1. Determine with justification the global behavior of $f(x)$.
2. Determine with justification the local behavior of $f(x)$:
   (a) Zeros and Multiplicities.
   (b) Vertical Asymptotes and potential sign changes.
3. Sketch $f(x)$, label the information from parts 1. and 2.

Exponential and Logarithmic Functions

1. Given the observation that 10 grams of Ce-137 decays to 7 grams in 14 hours, determine the rate of decay of Ce-137, the half-life of Ce-137, and how much of the original amount is present after 24 hours. *(Numbers are artificial.)*
2. Consider the Logistic Growth Model for the population of the Earth,

   $$A(t) = \frac{10}{1 + a \cdot e^{-0.0045 \cdot t}}$$

   where $A(t)$ is in billions of humans t years after the year 2000.

   (a) If the population of the Earth was approximately 6 billion in the year 2000, what is the value of a?
   (b) Based on this model, what is the maximum capacity for human life on of Earth? Justify your claim.
   (c) Suppose a new model is created under the assumption that the Earth can support 17 billion humans. Given the approximation of 6 billion humans on the Earth in the year 2000, determine the growth rate of the human population.

   (Numbers are artificial.)

Trigonometry

1. Graph a single cycle of the six trigonometric functions.
2. Consider the following equation.

   $$2\sin(2x - \pi) - \sqrt{2} = 0$$

   (a) Solve for *ALL* x values.
   (b) Which x values satisfy the equation in $[0, 2\pi]$?
3. Evaluate the following:

   (a) $\arccos\left(\cos\left(\frac{10\pi}{3}\right)\right)$ (b) $\tan\left(\arcsin\left(\frac{-3}{7}\right)\right)$
4. Using the sum and difference and Pythagorean identities, construct the Double Angle identities for sine and cosine.

   (a) $\sin(2\theta)$ (b) $\cos(2\theta)$