Algorithm verification: Sample problems

The problem below, with solution, is a model to help solving problems in assignment 2.

1. **Sum problem**

Find the sum of the elements on an array of numbers.

**Iterative solution with assertions:**

\[
\text{Sum}(A) > \text{Precondition: } A \text{ is an array of numbers}
\]

\[
i \leftarrow 1
\]

\[
y \leftarrow 0
\]

\[
\text{while } i \leq \vert A \vert
\]

\[
\]

\[
y \leftarrow y + A[i]
\]

\[
i \leftarrow i + 1
\]

return \( y \)


**Correctness proof:**

We proceed by induction.

1. (Base) The loop invariant holds on the first iteration, because \( y \) is 0 and \( A[i - 1] \) refers to no element of \( A \).
2. (Induction) If the loop invariant holds on the \( n \)th iteration, then it will hold on the \( (n + 1) \)st, because on each iteration \( A[i] \) is added to \( y \), and the element \( A[i] \) takes the name \( A[i - 1] \) after the incrementing of \( i \).
3. The precondition follows from the loop invariant, because at the beginning of the last iteration, \( i = \vert A \vert - 1 \), and that iteration adds 1 to \( i \) and adds \( A[\vert A \vert] \) to \( y \).
4. Hence at the end of the last iteration, \( y = A[1] + A[2] + \ldots + A[i] \), which is the same assertion as the postcondition, since \( i = \vert A \vert \).

2. **Last-zero problem**

**Problem:** Find the location of the rightmost zero in a bit array.

**Solution:**

\[
\text{Last0}(A)
\]

// Pre: \( \vert A \vert > 0 \) and \( A \) contains a 0

\[
i \leftarrow 1
\]

\[
\text{while } i \leq \vert A \vert
\]

> Loop invariant: \( y \) stores location of rightmost 0 in \( A[1 .. i - 1] \)

\[
\text{If } A[i] = 0
\]

\[
y \leftarrow i
\]

\[
i \leftarrow i + 1
\]

return \( y \)

// Post: \( y \) stores location of rightmost 0 in \( A \)

**Correctness proof:**

We proceed by induction. [To be completed.]