Finite transducers

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Definition

A *Mealy machine* is a 6-tuple \( \langle Q, \Sigma, \Gamma, \delta, \text{out}, q_0 \rangle \) where

- \( Q \) is a set of states, \( \Sigma \) a finite input alphabet, \( \delta \) a transition function, \( q_0 \) a start state as with DFAs
- \( \Gamma \) is a finite output alphabet
- \( \text{out} \) is an output function \( Q \times \Sigma \to \Gamma \)
- There is no final state because the transducer does not halt

Stream I/O

- Transducers such as Mealy machines model interactive devices such as ICs, controllers, etc.
- A Mealy machine computes a function \( \Sigma^\infty \to \Gamma^\infty \) where:
  \[ \Sigma^\infty = \{ ax \mid a \in \Sigma, x \in \Sigma^\infty \} \]
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- \( \Sigma^\infty \) is the set of *streams over* \( \Sigma \)

Example: soda machine

- Inputs (left side of labels): \( \{ Q, $1 \} \)
- Outputs (right side of labels): \( \{ “25”, “50”, “75”, [soda] \} \)

Example: digital clock

- Inputs: \{ tick \}
- Outputs: \( \{ "12:00:00", "12:00:01"… \} \)
- One state per time value
- With alarm, \( (24 \times 60 \times 60)^2 \) states

Other examples:

- Calculator
- Microprocessor
- Memory chip
- Any digital control device

Related ideas

- Transducers were once called “sequential machines” and were part of Curriculum 68, the first ACM CS curriculum
- Not part of standard theory texts today
- Related areas:
  - Markov decision processes,
  - model checking of reactive systems,
  - temporal logic