Teach What You Do: Providing an Authentic Mathematical Experience in an Introduction to Proofs Class

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What is Math?

Math is about more than writing proofs. It is about

- exploring
- looking for patterns
- guessing
- making mistakes
- learning about universal truths
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A bridge course should prepare students to succeed in mathematics. That means we need to prepare them to succeed in

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How do we Build that Bridge?

Provide activities that promote exploration
Reward taking chances
Don't penalize students’ mistakes (as long as they learn from them!)
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Points-Free Grading

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- Doesn’t penalize early mistakes
- Encourages taking risks
Sums of Consecutive Integers

- Calculate $29 + 30 + 31$.
- Calculate $21 + 22 + 23 + 24$.
- Add up the numbers from 16 to 20. That is, calculate $16 + 17 + 18 + 19 + 20$.
- Add up the numbers from 6 to 14.
- Calculate the sums of other sequences of consecutive integers.
- What pattern(s) do you observe?
- Can you extend any of the patterns?
- How would you describe or explain the patterns?
- Is there anything special about any of the numbers?
- What questions do the patterns raise for you? What else would you like to know?
Conjectures From Sums of Integers

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The Environment

Two Example Activities

Conclusions

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Conjectures From Sums of Integers

- The sum of any three consecutive integers is divisible by 3.
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- For any positive integer \( n \), the sum of any \( n \) consecutive integers is divisible by \( n \).
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- Any odd number can be written as a sum of consecutive integers.
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- The number 90 can be written as a sum of consecutive integers in exactly four different ways.
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- The number 90 can be written as a sum of consecutive positive integers in exactly four different ways.
Patterns in the Multiplication Table

Looking at the products in a $12 \times 12$ multiplication table:

- What patterns do you notice in how often numbers appear?
- Do you have any ideas about how you might predict how often a given number would appear in the table?
- What other questions do the patterns raise for you? What else would you like to know?
Conjectures From the Multiplication Table
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- The product of any pair of twin primes is always one less than a perfect square multiple of 36.
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- The product of any pair of twin primes is always one less than a perfect square multiple of 36.
- For any positive integer, \( n > 1 \), if the prime factorization of \( n \) is \( n = p_1^{e_1} \cdots p_k^{e_k} \), then \( n \) has \( (e_1 + 1) \cdots (e_k + 1) \) distinct positive integer factors.
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- Conjecturing activities help students see proofs as natural, rather than as rituals
- Creating good opportunities for conjecturing is hard but definitely worth it!
References
